LIMITS , CONTINUITY AND GRAPH THEORY

SOME OF THE STANDARD LIMITS:

1. $\lim\limits_{x\to a} \frac{x^n-a^n}{x-a} - na^{n-1}$; $\lim\limits_{x\to a} \frac{x^n-a^n}{x^m-a^m} = \frac{n}{m}a^{n-m}$

2. $a)$ $\lim\limits_{x\to 0} \frac{sinx}{x}$ =1= $\lim\limits_{x\to 0} \frac{tanx}{x}$ = $\lim\limits_{x\to 0} \frac{\sin^{-1}x}{x}$ = $\lim\limits_{x\to 0} \frac{tan^{-1}x}{x}$

   $b)$ $\lim\limits_{x\to 0} \frac{sinmx}{x}$ =m ; $\lim\limits_{x\to 0} \frac{sinmx}{sinnx} = \frac{m}{n}$

   c) $\lim\limits_{x\to 0} \frac{1-cosmx}{x^2}$ = $m^2/2$; $\lim\limits_{x\to 0} \frac{1-cosmx}{1-\cos nx}$ = $\frac{m^2}{n^2}$;

$\frac{\cos mx - cosnx}{x^2}$ = $\frac{n^2 - m^2}{2}$ where x is in radian measure.

3 $\lim\limits_{x\to 0}$.$(1+x)^{\frac{1}{x}}$ =e ; $\lim\limits_{x\to 0} (1+mx)^{\frac{1}{x}} = e^m$

4. $\lim\limits_{x\to\infty} \left(1+\frac{1}{x}\right)^x - e$; $\lim\limits_{x\to\infty} \left(1+\frac{m}{x}\right)^x - e^m$

5. $\lim\limits_{x\to 0} \left(\frac{e^x-1}{x}\right)$ =1 ; $\lim\limits_{x\to 0} \left(\frac{a^x-1}{x}\right)$ =$\log a$

6. $\lim\limits_{x\to 0} \frac{\log(1+x)}{x} = \log_e e = 1$

7. $\lim_{n\to\infty} \quad a^n = \begin{cases} 0 & if -1 < a < 1 \\ \infty & if \quad a > 1 \\ not\ defined & if a < -1 \end{cases}$

8. LHospital's Rule:

If f(x) and g(x) are differentiable functions at x=a such that f(a)=0=g(a) OR f(a)=∞ = g(a) then,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

(Note that numerator and denominator are to be differentiated separately)

;

Questions and solution

1. $\lim_{x\to 1} \frac{x^3-2x^2-x+2}{(x^2+x-2)}$ is

a) 1 b. 2/3 c. – 2/3 d. 3/2

SOLUTION: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$\lim_{x\to 1} \frac{3x^2-4x-1}{2x+1} = \frac{3-4-1}{2+1} = -\frac{2}{3}$

Ans: c

2. $\lim_{x\to 3} \left[\frac{1}{x-3} - \frac{3}{x(x^2-5x+6)}\right]$ is

a. 0 b. ¾ c. 4/3 d. - 4/3

SOLUTION: Given lim is = $\lim_{x\to 3} \left[\frac{1}{x-3} - \frac{3}{x(x-2)(x-3)}\right]$

$= \lim_{x\to 3} \frac{x^2-2x-3}{x(x^2-5x+6)}$ SOLUTION: ($\frac{0}{0}$ form) Using L.H.Rule

$= \lim_{x\to 3} \frac{2x-2}{(3x^2-10x+6)} = \frac{4}{3}$

Ans: c

3. $\lim_{x\to 0} \frac{1}{x} \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is

a. – 2 b. 2 c 0 d. ∞

SOLUTION: put x = tan θ As x$\to 0$, $\theta \to 0$

Given limit becomes $\lim\limits_{\theta\to 0} \frac{1}{\tan\theta} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \lim\limits_{\theta\to 0} \frac{2\theta}{\tan\theta}$ = 2

Ans. b

4. $\lim\limits_{x\to o} \frac{1-\cos 4x}{\cos 6x-\cos 4x}$ =

a. 4/5 b. – 4/5 c. 1 d. 8/5

Given expression is of the form $\lim\limits_{x\to 0} \frac{1-\cos mx}{\cos px-\cos qx}$ $=\frac{m^2}{q^2-p^2}$

$\therefore \lim\limits_{x\to o} \frac{1-\cos 4x}{\cos 6x-\cos 4x}$ $=\frac{4^2}{4^2-6^2}$ $=\frac{-4}{5}$

5. The least integer n for which

$\lim\limits_{x\to 0} \frac{e^x-\sin x-\cos x}{x^n}$ is finite and non zero is

a. 0 b. 1 c. 2 d . 3

SOLUTION: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$=\lim\limits_{x\to 0} \frac{e^x-\cos x+\sin x}{n\,x^{n-1}}$ ($\frac{0}{0}$ form) and lim =0 if n=1 $\therefore$ n$\neq 1$

$=\lim\limits_{x\to 0} \frac{e^x+\sin x+\cos x}{n(n-1)x^{n-2}}$ =2/2 is finite if n=2

Ans: c

6. $\lim_{\theta \to 0} \dfrac{sin^2\ \theta\ .\tan 4\ \theta}{tan2\theta^2\ \sin 3\theta}$ is

a. 2/3 b. 4/3 c. 3/4 d. 3/2

SOLUTION: Given lim is $= \lim_{\theta \to 0} \dfrac{\frac{sin^2\ \theta}{\theta^2} \times \theta^2\ .\frac{tan4\theta}{4\theta} \times 4\theta}{\frac{tan2\theta^2}{2\theta^2} \times 2\theta^2\ \frac{sin3\theta}{3\theta} \times 3\theta}$

$= \lim_{\theta \to 0} \dfrac{\theta^2\ . \times 4\theta}{2\theta^2 \times 3\theta} = 2/3$

Ans: a

7. $\lim_{x \to 2} \dfrac{\sqrt{2x^2 - 1} - \sqrt{3x+1}}{\sqrt{x^3+1} - \sqrt{2x+5}}$ is

a. $3/\sqrt{7}$ b. $2\sqrt{7}$ c. $3/2\sqrt{7}$ d. $3/4\sqrt{7}$

SOLUTION: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$= \lim_{x \to 2} \dfrac{\frac{1}{2\sqrt{2x^2 - 1}}(4x) - \frac{1}{2\sqrt{3x+1}}(3)}{\frac{1}{2\sqrt{x^3+1}}(3x^2) - \frac{1}{2\sqrt{2x+5}}(2)} = \dfrac{\frac{8}{2\sqrt{7}} - \frac{3}{2\sqrt{7}}}{\frac{12}{6} - \frac{2}{6}} = \dfrac{3}{2\sqrt{7}}$

Ans: c

8. $\lim_{n \to 0^+} \dfrac{1^2 + 2^2 + 3^2 + \text{---------} + n^2}{2n^3 + 3n^2 + 5}$ *is*

a. 1/ 12 b. 1/6 c. 2/3 d. 1/30

SOLUTION: Given lim is $= \lim_{n\to\infty} \frac{\frac{n(n+1)(2n+1)}{6}}{2n^3 + 3n^2 + 5}$

$$\lim_{n\to\infty} \frac{\frac{2n^3 + 3n^2 + n5}{6}}{2n^3 + 3n^2 + 5} = \frac{coefficient\ of\ highest\ power\ of\ n\ in\ Nr.}{coefficiant\ of highest\ power\ of\ n\ in\ Dr.}$$

$= \frac{1}{6} \times \frac{2}{2}$ =1/6 Ans: b

9. $\lim_{n\to\infty} [(2^n + 1)\ (7^n + 10^n)]^{\frac{1}{n}}$

a. 10/3 b. 10 c. 20 d. 30

SOLUTION: Given lim is = $\lim_{n\to\infty} \left[2^n \left(1 + \frac{1}{2^n}\right)\ (10)^n \left(\frac{7^n}{(10)^n} + 1\right)\right]^{\frac{1}{n}}$

$= \lim_{n\to\infty} [[2^n (10)^n]^{\frac{1}{n}} \left(1 + \frac{1}{2^n}\right)^{\frac{1}{n}} \left(\frac{7^n}{(10)^n} + 1\right)^{\frac{1}{n}}$ = $2\times 10 = 20$

Ans: c

10. $\lim_{x\to\infty} \sqrt{x}(\sqrt{2x+1} - \sqrt{2x-1})$ is

a. $2\sqrt{2}$ b. $\sqrt{2}$ c. $1/\sqrt{2}$ d. $-1/\sqrt{2}$

Solution: Given lim is

$$\lim_{x\to\infty} \sqrt{x}(\sqrt{2x+1} - \sqrt{2x-1}) \frac{(\sqrt{2x+1} + \sqrt{2x-1})}{(\sqrt{2x+1} + \sqrt{2x-1})}$$

$$= \lim_{x\to\infty} \frac{2\sqrt{x}}{(\sqrt{2x+1}+\sqrt{2x-1})} \quad \lim_{x\to\infty} \frac{2}{(\sqrt{2+\frac{1}{x}}+\sqrt{2-\frac{1}{x}})}$$

$$= \frac{2}{(\sqrt{2}+\sqrt{2})} = \sqrt{\frac{1}{2}}$$

Ans. c

b. $\lim_{x\to\infty} \left(\frac{x+4}{x+2}\right)^{x}$ is

a) e^2 b) e^6 c) e^3 d) 0

SOLUTION: Given lim is $= \lim_{x\to\infty} \left(\frac{x+2+2}{x+2}\right)^{x} = \lim_{x\to\infty} \left(1+\frac{2}{x+2}\right)^{x}$

$$= \lim_{x\to\infty} \left[\left(1+\frac{2}{x+2}\right)^{\frac{x+2}{2}}\right]^{\frac{2}{x+2}\times x} = \lim_{n\to\infty} e^{\frac{2x}{x+2}} = e^2 \text{ ans a}$$

Ans: a

c. $\lim_{x\to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$ is

a. $\sqrt{2/3}$ b. 2/3 c. 4/9 d. 8/27

SOLUTION: Given lim is $\left(\frac{2}{3}\right)^{\left(\frac{0}{0}\text{ form}\right)} = \lim_{x\to 1} \left(\frac{1+x}{2+x}\right)^{\frac{1}{1+\sqrt{x}}}$

$$= \left(\frac{2}{3}\right)^{\frac{1}{2}} = \sqrt{\frac{2}{3}}$$

Ans: a

d. $\lim_{x\to 0^+} \dfrac{a^{tanx} - a^{\sin x}}{\tan x - \sin x}$ is

a. log a b. – log a c. tan x d. 1

SOLUTION: As x→ 0^+ (tanx-sinx) → 0^+

Given lim is = $\lim_{x\to 0^+} \dfrac{(a^{tanx-sinx} - 1)a^{sinx}}{\tan x - \sin x}$

$= \lim_{(\tan x - sinx)\to 0^+} \dfrac{a^{tanx-sinx} - 1}{\tan x - \sin x} = loga$ Ans: a

e. The value of f(0) , so that the function f (x) = $\dfrac{2x - \sin^{-1}x}{2x + \tan^{-1}x}$

is continuous at each point in its domain, is

a.– 1/3 b. 0 c. 1/3 d. 3

14. SOLUTION: Function is continuous at x = 0

$\therefore \lim_{x\to 0} f(x) = f(0)$

$$= \lim_{x\to 0} \frac{2x - \sin^{-1}x}{2x + \tan^{-1}x} = \lim_{x\to 0} \frac{2 - \frac{\sin^{-1}x}{x}}{2 + \frac{\tan^{-1}x}{x}} = \frac{2-1}{2+1} = 1/3$$

Ans : c

f. If function f(x) = $\begin{cases} \frac{1-\cos 4x}{x^2} & for\ x < 0 \\ m & for\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & for\ x > 0 \end{cases}$

Is continuous at x = 0 then the value of m is

a. 0 b. 2 c. 4 d. 8

SOLUTION: Function is continuous at x = 0

$\therefore \lim_{x\to 0} f(x)\ exists\ \therefore\ L.H.L = R.H.L$ =f(0)=m

$\therefore$ L. H.L. = $\lim_{x\to 0} \frac{1-\cos 4x}{x^2}$ = m

i.e. $\lim_{x\to 0} \frac{1-\cos 4x}{x^2}$ = 16/2 =8 (using $\lim_{x\to 0} \frac{1-\cos mx}{x^2}$ = m^2 / 2)

Ans. d

g. if f (x) = $\begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Then which of the following is true.

a. Left hand limit = Right hand limit b. Limit does not exist

c. f(x) is continuous at x = 0 d. f(x) is differentiable at x = 0

Solution: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0} \frac{-x}{x} = -1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} \frac{+x}{x} = +1$

Using $|x| = \begin{cases} -x & if\ x < 0 \\ x & if\ x \geq 0 \end{cases}$

∴ R. H.L ≠ L.H.L   Limit does not exist.   Ans. b

h. The function f(X) = $\begin{cases} 2x - 1 & if \quad x < -1 \\ 3x^2 + 1 & if\ -1 \leq x < 3 \\ x^3 + 1 & if \quad 3 \leq x < 4 \end{cases}$ Is discontinuous at

a. -1   b. 3   c. -1, 3   d. none of these

Solution: We have to check the continuity at x = -1 , 3

$\lim_{x \to -1} f(x) = \lim_{x \to -1} 2x - 1 = -3$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1} 3x^2 + 1 = 4$

∴ R. H.L ≠ L.H.L ∴ f(x) is a discontinuous at – 1

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3} 3x^2 + 1 = 28$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3} x^3 + 1 = 28$

∴ R. H.L = L.H.L = f(3) ∴ f(x) is a continuous at 3

∴ Ans. a.

i. If f(x) = $\begin{cases} \dfrac{x\, e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} & x \neq 0 \\ \dfrac{k}{2} & x - 0 \end{cases}$

Is continuous at x = 0 then the value of k is

a.-1 b. 2 c. 0 d. 1

Solution : $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x\, e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$

$= \lim_{x \to 0} \frac{x}{e^{\frac{-1}{x}}+1} = \frac{0}{0+1} = 0 \Rightarrow \frac{k}{2} = 0 \quad \therefore k = 0$ Ans. c.

19. Which of the following is true always

a. If f(x) is continuous at x = a then it is differentiable at x = a

b. If f(x) and g(x) are continuous at x = a

Then(f(x) – g(x)) need not be continuous at x=a

c. Every polynomial function is continuous in the region (- ∞, ∞)

d. None of these.

Ans. c.

20. Let f(x) = $[x] + \lfloor -x \rfloor$ where [ ] denotes greatest integer part then for any integer m

a. f(x) is continuous at x = m

b. $\lim_{x \to m} f(x)$ exists but ≠ f (m)

c. $\lim_{x \to m} f(x)$ does not exists

d. f (x) is differentiable at x = m

Solution:We have to check the continuity for x $< m$ and x $> m$

If x<m $[x] = m - 1,$ $[-x] = -m$

$\therefore \lim_{x \to m^-} f(x) = m - 1 - m = -1$

If x > m $[x] = m$, $[-x] = -m$ - 1

$\therefore \lim_{x \to m^+} f(x) = m + (-m - 1) = -1$

f (m) = m – m = 0

$\therefore \lim_{x \to m} f(x)$ exists but ≠ f (m) *Ans. b*

21. If f(x)= $\begin{cases} \frac{\sqrt{1+mx}-\sqrt{1-mx}}{x}, & -1 \le x < 0 \\ \frac{2x-1}{x-2} & 0 < x \le 1 \end{cases}$ is continuous in [-1 1]

Then the value of m is

a. 2 b. ½ c. – ½ d. -2

Solution : f (x) is continuous in [-1 , 1] ∴at 0

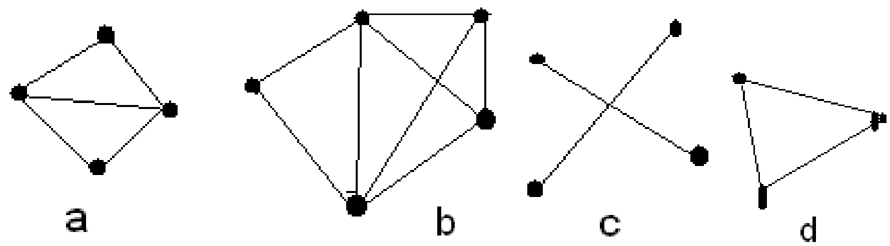
$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} \frac{\sqrt{1+mx}-\sqrt{1-mx}}{x}$ ($\frac{0}{0}$ form) Using L.H.Rule

$\lim_{x \to 0} \frac{\frac{1}{2\sqrt{1+mx}} m - \frac{1}{2\sqrt{1-mx}}(-m)}{1} = \frac{m}{2} + \frac{m}{2} = m$

$\lim_{x \to 0^+} f(x) = \frac{-1}{-2} = \frac{1}{2}$ ∴ m=1/2 Ans: b

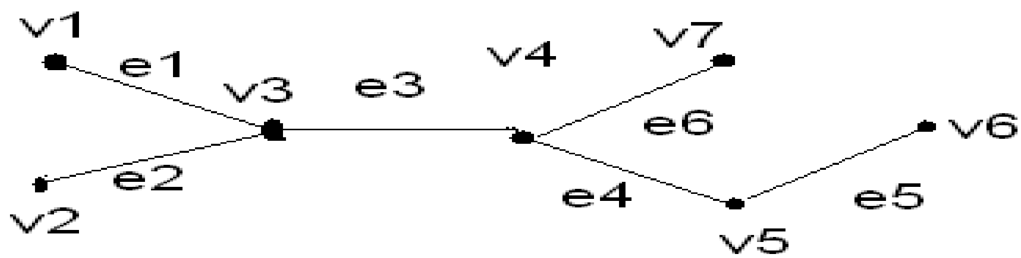
22. Which of the following is an Euler graph?



Ans: d.

Solution: A graph in which the degree of every vertex is even is called Eulerain graph

23. If **m** is the number of cut vertices and **n** is the number of bridges in a given graph then **m+n** is



a. 11 b. 6 c. 7 d. 9

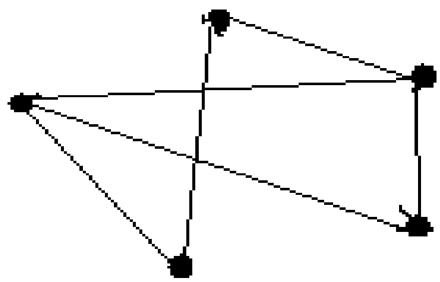
Solution: A vertex V of a graph G is calledcut vertex if its removal Increases the number of components. Here V3 ,V4,V5 are Cut vertices.

Bridge is an edge of a graph whose removal increases the number of components. Here all the 6 edges are bridges.

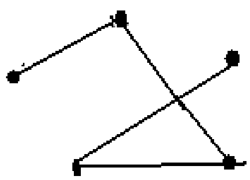
NOTE: Every edge of a tree is a bridge.

Ans: d

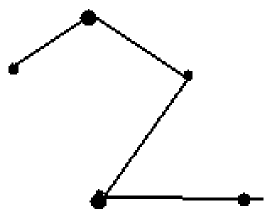
24. The compliment of the given graph



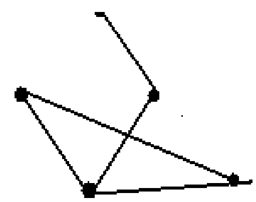
is



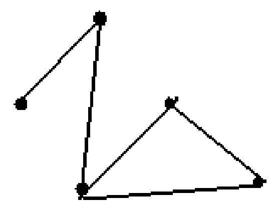
a



b



c

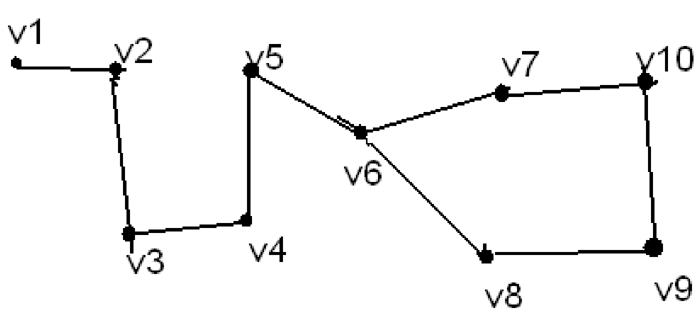


d

Solution: Compliment of a graph G is the graph obtained by deleting the edges of G in the complete graph of G

Ans: a

25. The length of longest cycle and longest path for a given graph



a) (5,10) b)(6,10) c) (5,9) d) (6,8)

Solution: A walk in which no vertices and no edges are repeated is called path.

Here length of path is 9

A closed walk in which all vertices and intermediate vertices are distinct is called Cycle

$V_6 V_8$, $V_8 V_9$, V_9 V10, $V_{10} V_7$, $V_7 V_6$ –5 edges

Ans: c

26. If sum of the degrees of all Vertices of a graph **G** is 24, then the number of edges in that graph is

a. 23. b. 10 c. 6 d. 12

Solution: The sum of the degrees of all vertices in a graph G

Is twice the number of edges in G

∴ Ans: d

27. The number of edges in a complete graph of **'n'** vertices is

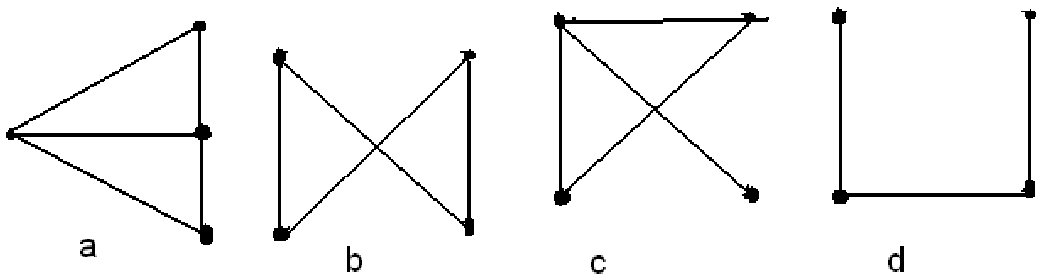
a. n b. n(n-1) c. $\frac{n(n-1)}{2}$ d. n/2

A simple graph in which there exists an edge between every pair of vertices is called complete graph.

Therefore number of edges $^nC_2$

Ans: c

28. Which of the following is a bipartite graph ?



Solution: A graph G is called bipartite graph if every edge of G connects a vertex set V_1 to vertex set V_2 where V_1, V_2 are dis-joint subsets of V

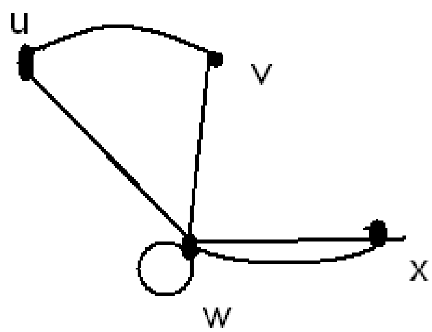
Ans: b

29. If F, V, E denote faces ,vertices, and edges of polyhedron respectively then Euler's formula is

a. F+E = V+2
b. F+V = E+2
c. V+E = F+2
d. F+V = 2 E

Ans: b

30.



In the above graph degree of w is

a.4 b. 5 c. 6 d. 7

Solution: The degree of vertex is the number of edges incident on it with loops counted twice Ans: c

31. A graph is called psuedograph if

a. It has no loops and no multiple edges.
b. It has multiple edges and no loops.
c. It has both multiple edges and loops.
d. It is **a** tree.

Ans: c

32. An isolated vertex is

a. Not a cut vertex.
b. A cut vertex.
c. A complete graph.
d. Of degree one

Ans: A vertex of a graph which is not an end vertex of any edge is called isolated vertex .Degree of isolated vertex is zero.

33. If f (a) = 3 , g (a) = 2 , f ‘ (a) = 1 and g ‘ (a) = - 1

Then $\lim_{x \to a} \frac{f(x)g(a) - f(a).g(x)}{x-a}$ is

a.0 b. 5 c. – 5 d . 6

SOLUTION: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$$=\lim_{x \to a} \frac{f'(x)g(a) - f(a).g'(x)}{1} = f'(a)g(a) - f(a).g'(a)$$

= 2-3(-1) =5

Ans: b

34. $\lim_{x \to 1} \frac{\sum_{m=1}^{100} x^m - 1}{x-1}$ is

a)1050 b) 5050 c) 1010 d) 5010

Solution: Given lim =$\lim_{x \to 1} \left(\frac{x+x^2+x^3+\cdots+x^{100}-100}{x-1} \right) \left(\frac{0}{0} form \right)$

Using L.H rule

$$= \lim_{x \to 1} \frac{1+2x+3x^2+\cdots+100\,x^{99}}{1}$$

=1+2+…+100=50×101=5050

Ans: b

35. $\lim_{x\to\frac{\pi}{3}} \frac{tan^3x-3tanx}{\cos(x+\frac{\pi}{6})}$ is

a.12 b)24 c)-12 d)-24

SOLUTION: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$$= \lim_{x\to\frac{\pi}{3}} \frac{3tan^2xsec^2x-3sec^2x}{-\sin(x+\frac{\pi}{6})} = \frac{3\times3\times4-3\times4}{-1} = -24$$

Ans: d

36. $\lim_{x\to1} \frac{x^{\frac{1}{3}}+x^{\frac{1}{4}}-2}{x^3-1}$ is

a. 7/36
b. + 1/36
c. – 1/12
d. 7/12

Solution: Given lim is = $\lim_{x\to1} \frac{x^{\frac{1}{3}}-1+x^{\frac{1}{4}}-1}{x^3-1}$

$$= \lim_{x\to1} \frac{x^{\frac{1}{3}}-1}{x^3-1} + \lim_{x\to1} \frac{x^{\frac{1}{4}}-1}{x^3-1} = \frac{\frac{1}{3}}{3} + \frac{\frac{1}{4}}{3} = \frac{1}{9} + \frac{1}{12} = \frac{7}{12}$$

using $\lim_{x\to a} \frac{x^m-1}{x^n-1}$ =m/n

Ans: d

37. $\lim_{n\to\infty} \frac{2^{-n}(n^2+5n+6)}{(n+5)(2n-1)}$ is

a. 0
b. 1
c. ∞
d. - 2

Solution: As n→ ∞ $2^{-n} \to 0$ Hence $\lim_{n\to\infty} \frac{2^{-n}(n^2+5n+6)}{(n+5)(2n-1)}$ =0× $\frac{\infty}{\infty}$ =0

Ans: a

38. $\lim_{\theta\to\frac{\pi}{4}} \frac{\sqrt{2}-cos\theta-sin\theta}{(4\theta-\pi)^2}$ is

a. 1/32
b. 1/16
c. $\frac{\sqrt{2}}{16}$
d. $\frac{\sqrt{2}}{32}$

Solution: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$= \lim_{\theta\to\frac{\pi}{4}} \frac{sin\theta-cos\theta}{2(4\theta-\pi)\times 4}$ ($\frac{0}{0}$ form) = $\lim_{\theta\to\frac{\pi}{4}} \frac{+cos\theta+sin\theta}{32}$

$= \frac{1}{32} \times \frac{2}{\sqrt{2}}$ = $\sqrt{2}$/32

Ans: d

39. $\lim_{x\to 0} \frac{e^x-e^{xcosx}}{x+sinx}$ is

a. 0

b. 1

c.2

d.- 1

Solution: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$$= \lim_{x\to 0} \frac{e^x - e^{x\cos x}(-x\sin x + \cos x)}{1+\cos x} = \frac{1-1(0+1)}{2} = 0$$

Ans: a

40. $\lim_{x\to 0} \left(\frac{1-\tan x}{1-\sin x}\right)^{\operatorname{cosec} x}$ is

a. 0

b. 1

c. e

d. 1/e

Solution: $\lim_{x\to 0} \left(\frac{1-\tan x}{1-\sin x}\right)^{\operatorname{cosec} x} = \lim_{x\to 0} \left[\frac{(1-\tan x)^{\frac{-1}{\tan x}\times -\tan x \operatorname{cosec} x}}{(1-\sin x)^{-\frac{-1}{\sin x}}}\right]$

$\lim_{x\to 0} \frac{e^{-\tan x \operatorname{cosec} x}}{e^{-1}} = \lim_{x\to 0} \frac{e^{-\sec x}}{e^{-1}} = \frac{e^{1}}{e^{-1}}$ =1

Ans:b

41. $\lim_{x\to 0^+} \frac{\sin\sqrt{x}}{\sqrt[4]{x}}$ is

a. 1

b. 0

c. – 1

d. – ¼

Solution:

Given lim is $\lim_{x\to 0^+} \frac{\sin\sqrt{x}}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt[4]{x}} = \lim_{x\to 0^+} 1 \times \frac{\sqrt{x}}{\sqrt[4]{x}} = \lim_{x\to 0^+} \sqrt[4]{x} = 0$

42. $\lim\limits_{n\to\infty} \frac{3(2)^{n+1}-4(5)^{n+1}}{5(2)^{n}+8(5)^{n}}$ is

a. – 5/2

b. 5/2

c. 10

d. – ¼

Solution: Given lim is $\lim\limits_{n\to\infty} \frac{3(\frac{2}{5})^{n+1}-4}{5(\frac{2}{5})^{n}+8} \times \frac{5^{n+1}}{5^{n}} = \frac{5(3\times0-4)}{(5\times0+8)}$

43. $\lim\limits_{x\to 0} \frac{(5)^{x}+(5)^{-x}-2}{(x)^{2}}$ is

a. 2log5

b. $(log5)^2$

c. 0

d. 1

Solution: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$$\lim_{x\to 0} \frac{(5)^{x}\log 5-(5)^{-x}\log 5}{2x} = \lim_{x\to 0} \frac{(5)^{x}(\log 5)^{2}+(5)^{-x}(\log 5)^{2}}{2}$$

$$= \lim_{x\to 0}(5)^{x}(\log 5)^{2} = (\log 5)^{2}$$

Ans. b

44. $\lim\limits_{n\to\infty}\left[\frac{1}{1-n^{2}}+\frac{2}{1-n^{2}}+\frac{3}{1-n^{2}}+\cdots+\frac{n}{1-n^{2}}\right]$ is

a)0 b)4 c)1/2 d)-1/2

Solution: Given lim is = $\lim_{n\to\infty}$

$$\frac{1+2+3+4+\cdots+n}{1-n^2} = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{1-n^2} = \lim_{n\to\infty} \frac{\frac{\left(1+\frac{1}{n}\right)}{2}}{\left(\frac{1}{n^2}-1\right)} = -1/2$$

Ans: d

45. $\lim_{x\to 0} \frac{2^x+3^x-5^x-7^x}{\tan x}$ is

a) $\log_e 6/5$ b) $\log_e 6/35$ c) $\log_e 5/6$ d) $\log_e 6/15$

Solution: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$$=\lim_{x\to 0} \frac{2^x \log 2+3^x \log 3-5^x \log 5-7^x \log 7}{\sec^2 x} = \frac{\log 2+\log 3-\log 5-\log 7}{1}$$

=log6/35

46. $\lim_{x\to\infty} \frac{(x)^n}{(e)^x}$ $for\ all\ n \in N\ \ is$

a. e

b. n!

c. 0

d. 1/e

Solution: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule repeatedly

$$\lim_{x\to\infty} \frac{(x)^n}{(e)^x} = \lim_{x\to\infty} \frac{n(n-1)(n-2)(n-3)(n-4)\ldots\ 3\times 2\times 1}{(e)^x} = \frac{n!}{\infty} = 0$$

Ans. c

47. $\lim_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{2x+1}$ is

a. e^4

b. e^{-4}

c. e^6

d. e^2

Solution: Given lim is $= \frac{\lim_{x\to\infty}\left[(1+\frac{5}{x})^{\frac{x}{5}}\right]^{\frac{5}{x}\times\frac{2x+1}{1}}}{\lim_{x\to\infty}\left[(1+\frac{2}{x})^{\frac{x}{2}}\right]^{\frac{2}{x}\times 2x+1}} = \lim_{n\to\infty} \frac{(e)^{(10+\frac{5}{x})}}{(e)^{(4+\frac{2}{x})}}$

$\frac{e^{10}}{4} = e^6$

Ans. c

48. $\lim_{\theta\to\frac{\pi}{6}} \frac{2\sin^2\theta+\sin\theta-1}{2\sin^2\theta-3\sin\theta+1}$ *is*

a)0 b)-1 c)1 d)-3

Solution: Given lim is ($\frac{0}{0}$ form) put sin$\theta = x$ then $x \to \frac{1}{2}$

. $\lim_{\theta\to\frac{\pi}{6}} \frac{2\sin^2\theta+\sin\theta-1}{2\sin^2\theta-3\sin\theta+1}$=. $\lim_{x\to\frac{1}{2}} \frac{2x^2+x-1}{2x^2-3x+1} = \lim_{x\to\frac{1}{2}} \frac{(2x-1)(x+1)}{(2x-1)(x-1)}$

$= \frac{\frac{1}{2}+1}{\frac{1}{2}-1}$ =-3

Ans. d

49. $\lim_{x\to 1}\left(\frac{3}{1-x^3}-\frac{5}{1-x^5}\right)$ is

a)0 b)1 c)-1 d)3

Solution: Given lim is $=\lim_{x\to 1}\left(\frac{3-3x^5\ -5+5x^3}{(1-x^3\ )(1-x^5\ )}\right)\left(\frac{0}{0}\right.$ form) Using L.H.Rule

$$=\lim_{x\to 1}\left(\frac{-15x^4\ +15x^2}{-3x^2\ -5x^4\ +8x^7}\right)\left(\frac{0}{0}\ \text{form}\right)$$

$$=\lim_{x\to 1}\left(\frac{-60x^3\ +30x^2}{-6x^2-20x^3\ +56x^6}\right)\ =\frac{-30}{30}=-1$$

Ans. c

50. $\lim_{x\to\infty}\frac{(2x+3)^{40}(4x-1)^{10}}{(2x-5)^{50}}$ is

a)1 b) 2^{10} c) 2^5 d) 2^2

Solution: Given lim is $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\frac{coefficient\ of\ highest\ power\ of\ x\ in\ Nr.}{coefficiant\ of highest\ power\ of\ x\ in\ Dr.}$

$$=\frac{2^{10}\ \times 4^{10}}{2^{50}}=\frac{2^{60}}{2^{50}}=2^{10}$$

Ans. b

51. $\lim_{n\to\infty}\left(\frac{1+3+5+\cdots\ \ +n\ terms}{2+4+6+\cdots .+n\ terms}\right)^n$ is

a) 1 b) 2 c) 1/e d) e

Solution: Given lim = $\lim_{n\to\infty}\left(\frac{n^2}{n(n+1)}\right)^n = \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)^n = \frac{1}{e}$

Ans: c

52. $\lim_{x\to 0}\frac{\tan\sqrt[3]{x}\,.\log(1+3x)}{(\tan^{-1}\sqrt{x})^2\,(e^{\sqrt[3]{x}}-1)}$ is

a. ½ b. 3 c.0 d.1/3

Solution : Given lim = $\frac{\lim_{x\to 0}\left(\frac{\tan\sqrt[3]{x}}{\sqrt[3]{x}}\ \sqrt[3]{x}\right)\ \left(\frac{\log(1+3x)}{3x} X\ 3x\right)}{\lim_{x\to 0}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2\times x\ \left(\frac{e^{\sqrt[3]{x}}-1}{\sqrt[3]{x}}\right)\times\sqrt[3]{x}}$

$=\lim_{x\to 0}\frac{1\times\sqrt[3]{x}\times 1\times 3x}{1\times x\times 1\times\sqrt[3]{x}} = 3$

Ans: b

53. $\lim_{\theta\to 0}\frac{\sin\theta^2(1-\cos\theta^2)}{\theta^6}$ is

a. 1/2 b.1/4 c.0 d.1/3

Solution: $\lim_{\theta\to 0}\frac{\sin\theta^2}{\theta^2}\ \frac{(1-\cos\theta^2)}{(\theta^2)^2}$ =1X $\lim_{\theta\to 0}\frac{(1-\cos\theta^2)}{\theta^2}$ =1/2

Ans: a

54. $\lim_{x\to\frac{\pi}{4}}\frac{1-\cot x}{2-\operatorname{cosec}^2 x}$ is

a. -½ b. 1 c.1/4 d.1/2

SOLUTION: Given lim is ($\frac{0}{0}$ form) Using L.H.Rule

$$\lim_{x \to \frac{\pi}{4}} \frac{cosec^2 x}{2cosec^2 x \ cot x} = \frac{1}{2\cot\frac{\pi}{4}} = \frac{1}{2}$$

55. If f(x) $= \begin{cases} \frac{x^2 - c^2 x + 2c}{2x^2 - 5x + 2} & x \neq 2 \\ 1 & x = 2 \end{cases}$

Is continuous at x = 2 then the value of c

a. ± 1 b. 1 c. – 1 d. 0

Solution : Function is continuous at x = 2 for x = 2

Denominator is equal to zero ∴ numerator = 0 at x = 2

∴ $x^2 - c^2 x + 2c$ = 0 at x = 2

i.e. 4 - 2 c^2 + 2c = 0 i.e (c-2)(c+1)= 0

c=-1or2 c $\neq 2$ as

56. The function f(x) = [x] + $|1 - x|$ where [x] greatest integer x is

a. Continuous at x = 1
b. Dis continuous at x = 1
c. Limit as x --->1 does not exist

d. Derivable at x = 1

56. Solution: If x --->1 [x] = 0 $|1 - x|$ = 1- x [∵(1– x) > 0]

∴ $\lim_{x \to 1^-}$ f(x) = $\lim_{x \to 1}$ (0 + 1 – x) = 0

If x ---> 1^+ [x] = 1 $|1 - x|$ = x – 1 [∵ (1– x) < 0]

∴ $\lim_{x \to 1^+}$ f(x) = $\lim_{x \to 1}$ 1 + (x-1) = $\lim_{x \to 1} x$ = 1

∴ L.H. limit ≠ R.H.limit

Limit does not exist

Ans. c

57. If f (x) = $\begin{cases} \frac{\sin 3x}{x^3+4x} & x \neq \\ \frac{k}{2} & x = 0 \end{cases}$

Is continuous at x = 0 then value of k is

a. ¾ b. 4/3 c. 3/2 d. 2/3

57. Solution : Function is continuous at x = 0

∴ $\lim_{x \to 0}$ f(x) = f(0)

$\lim_{x \to 0}$ f(x) = $\lim_{x \to 0} \frac{\sin 3x}{x^3+4x}$ = $\lim_{x \to 0} \frac{\sin 3x}{3x\,(x^2+4)}$ x 3

$$= \frac{1x\ 3}{0+4} = 3/4$$

Ans. a

58. Which of the following is false statement

a. If f(x) is continuous at x = a then $\lim_{x\to a}$ f(x) exists.

b. If f' (a) exists then f(x) is continuous at x = a

c. If f(x) is continuous at x = a the f'(a) exists

d. If $\lim_{x\to a}$ f(x) = f(a) f(x) is continuous at x = a

58.Solution: If function is differentiable then it is continuous but converse may not be true.

∴ Ans. c

59. The function f(x) = $\frac{1-\sin x+\cos x}{1+\sin x+\cos x}$ is not defined at x = Π The value of f(Π) so that f(x) is continuous at x = Π is

a. – ½ b. ½ c. -1 d. 1

59. Solution : Function is continuous at x = Π

∴ $\lim_{x\to\Pi}$ f(x) = f(Π)

∴ $\lim_{x\to\Pi}$ f(x) = $\lim_{x\to\Pi} \frac{1-\sin x+\cos x}{1+\sin x+\cos x}$ (0/0 form)

$$= \lim_{x\to\Pi} \frac{-\cos x-\sin x}{\cos x-\sin x} = \frac{-(-1)-0}{-1-0} = -1$$

Ans. c.

60. If f(x)=$\begin{cases} \frac{\sin(e^{x-2}-1)}{\log(x-1)} & when\ x \neq 2 \end{cases}$ is continuous at x=2 then f(2) is

a)e b)e^2 c)-e d)1

Solution: Function is continuous at x = 2

Given lim is = $\lim_{x\to 2} \frac{\sin(e^{x-2}-1)}{\log(x-1)}$

$$= \lim_{(x-2)\to 0} \frac{\frac{\sin(e^{x-2}-1)}{e^{x-2}-1}\times(e^{x-2}-1)}{\frac{\log(1+(x-2))}{x-2}\times(x-2)} = \qquad = \lim_{(x-2)\to 0} \frac{e^{x-2}-1}{x-2}$$

=1

Ans: d

61. If f(x)=$\begin{cases} \frac{\log_e(1+2x)-\log_e(1-2x)}{x} & if x \neq 0 \\ e^n & if\ x = 0 \end{cases}$

is continuous at x=0.then the value of n is

a) 4 b)e^4 c) $\log_e 4$ d) $\log_4 e$

Ans: Function is continuous at x=0

$\therefore \lim_{x\to 0}$ f(x) = f(0) =e^n

$$\lim_{x\to 0} \frac{\log_e(1+2x)-\log_e(1-2x)}{x} = \lim_{x\to 0} \frac{\log_e(1+2x)}{2x}\times 2 - \frac{\log_e(1-2x)}{-2x} \times -2$$

=2-(-2) =4

$\therefore\ e^n$=4 i.e n=$\log_e 4$

Ans: c

62. If f(x) $=\begin{cases} \frac{\sin[x]}{[x]} & [x] \neq 0 \\ 0 & [x] = 0 \end{cases}$

where [ ] denotes greatest integer part

$\lim_{x\to 0}$ f(x) is equal to

a) 1 b)-1 c)0 d)none of these

As $x \to 0^-$ $[x] = -1$ $\therefore$sin $[-1]$ = -sin1

$\lim_{x\to 0^-}$ f(x) = $\frac{-\sin 1}{-1}$ =sin1

As $x \to 0^+$ $[x]$ = 0 $\therefore$sin $[x]$ =sin0 = 0

$\lim_{x\to 0^+}$ f(x) = $\lim_{x\to 0} \frac{\sin[x]}{[x]}$ = 1

$\therefore L.H.L. \neq R.H.L.$

Ans: d

63. Which of the following is a False statement?

a)In a graph, every edge of a tree is abridge .

b)In a graph Edge set E can be an empty set .

c)Every graph must have even number of vertices of odd degree.

d)In any tree there must be at least one pendent vertex.

Solution: In a tree there must be minimum TWO pendent vertices.

Ans: d

64. In a complete K_n regular graph the degree of each vertex is

a)n(n-1)/2 b)n/2 c)n-1 d)n

Ans: c

65. The number of edges and vertices of K_5 graph is

a) 15, 5
b) 15 , 4
c) 10 , 4
d) 10, 5

Solution: number of edges of K_5 = $\frac{5(5-1)}{2}$ = 10

number of verticesof K_5 =5

Ans: d

66. The point of discontinuity of the function

f(x) = $\lim_{n\to\infty}\left(\frac{4^n\,(sin^2x)^n}{3^n-(4cos^2x)^n}\right)$ is

a) n$\pi \pm \frac{\pi}{3}$ b) n$\pi \pm \frac{\pi}{6}$ c) n$\pi \pm \frac{5\pi}{6}$ d) n$\pi \pm \frac{2\pi}{3}$

Solution: For all n $4^n\,(sin^2x)^n$ is continous

Function f(x) is dis continuous if $3^n-(4cos^2x)^n$=0

i.e. $3^n = 4^n(cos^2x)^n$ $\therefore (cos^2x)^n$ =$\frac{3^n}{4^n}$

$\therefore$ cos^2x= 3/4 $\Rightarrow x = \text{n}\pi \pm \frac{\pi}{6}$

Ans: b

67. $\lim_{x\to0}\left[\frac{|sinx|+|sin^2x\,|+|sin^3x\,|...\infty}{x}\right]$ is

a)1 b)0 c)2 d)-1

34. Solution: $|sinx|, |sin^2x|, |sin^3x|$ ...are in G.P

with $|sinx| < 1$, a= sinx and r= sinx

hence given limit $=\lim_{x\to 0}$ S∞$=\frac{a}{1-r}=\lim_{x\to 0}\frac{sinx}{x(1-sinx)}$

$=\lim_{x\to 0}\frac{sinx}{x}\lim_{x\to 0}\frac{1}{1-sinx}$ =1 Ans: a

68. $\lim_{x\to 0}\frac{xtan2x-2xtanx}{(1-cos2x)^2}$ is

a)2 b)-2 c)1/2 -d)-1/2

Solution: given lim$=\lim_{x\to 0}\frac{x\left[\frac{2tanx}{1-tan^{2x}}-2tanx\right]}{4\,(sin^2x)^2}$ $=\lim_{x\to 0}\left[\frac{x\,2tan^3x}{4sin^4x\,(1-tan^2x)}\right]$

$\lim_{x\to 0}\left[\frac{2\,x}{4sinx\,cos^3x\,\,(1-tan^2x)}\right]$ =1/2

Ans: a

69. $\lim_{x\,\,\,0}\left[\frac{e^{\frac{1}{x}}-1}{(1+e^{\frac{1}{x}})}\right]$ is

a) 1 b)0 c)does no exists d)none of these

Solution: As x$\to 0^+$ 1/x$\to\infty$ $e^{\frac{1}{x}}\to\infty$

$$\lim_{x\to 0^+}\left[\frac{e^{\frac{1}{x}}-1}{(1+e^{\frac{1}{x}})}\right]=\lim_{x\to 0}\left[\frac{1-\frac{1}{e^{\frac{1}{x}}}}{(1+\frac{1}{e^{\frac{1}{x}}})}\right]=1$$

As x › 0^- 1/x › ∞ $e^{\frac{1}{x}}$ › 0

$$\lim_{x\to 0^-}\left[\frac{e^{\frac{1}{x}}-1}{(1+e^{\frac{1}{x}})}\right] = \lim_{x\to 0^-}\left[\frac{e^{\frac{1}{x}}-1}{(1+e^{\frac{1}{x}})}\right] = \frac{0-1}{1+0} = -1$$

R.H.L.≠L.H.L. Limit does not exists

70. $\lim_{n\to\infty}\left[1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots n\ terms\right]$ is

a)2/3 b)3/5 c)-3/5 d)5/3

Solution: Let $S_n = 1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\cdots n\ terms$

$=1+\frac{-2}{3}+\left(\frac{-2}{3}\right)^2+\left(\frac{-2}{3}\right)^3+\cdots+$n terms is a G.P.

Where a=1 r=-2/3 hence $S_\infty = \frac{a}{1-r} = \frac{1}{1+\frac{2}{3}}$ =3/5