CIRCLES

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DEFINITIONS AND SOME IMPORTANT RESULTS

Definition of Circle : A circle is the locus of a point which moves such that its distance from fixed point is always a constant. The fixed point is called centre and the constant distance is called radius.

1. Equation of circle with centre at origin and radius '*a*' is given by $x^2 + y^2 = a^2$

2. Equation of the circle with centre at (h, k) and radius '*r*' is given by $(x-h)^2+(y-k)^2=r^2$

3. The standard equation of a circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $\equiv (-g, -f)$ & radius $(r) = \sqrt{g^2 + f^2 - c}$

If $g^2 + f^2 - c > 0$ circle is a real circle

If $g^2 + f^2 - c = 0$, circle is a point circle

If $g^2+f^2-c<0$, circle is an imaginary circle.

If the above circle touches X–axis then $g^2=c$

If the above circle touches Y–axis then $f^2=c$

If the above circle touches both axes then $g^2=f^2=c$

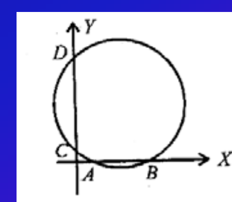
4. Equation of the circle described on the line joining (x_1, y_1) and (x_2, y_2) as a diameter is given by

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

5. i. The length of the chord cut off along the X-axis by the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

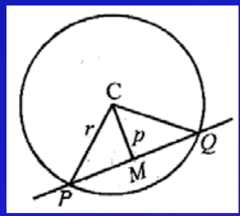
$$AB = 2\sqrt{g^2 - c}$$



ii. The length of the chord off along the Y-axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$CD = 2\sqrt{f^2 - c}$$

iii. The length of chord cut off by the line $ax + by + c = 0$ is given by



$$PQ = 2\sqrt{r^2 - p^2}$$

6. Equation of a circle in parametric form :

i. The equation of circle with centre at origin and radius equal to r in parametric form are $x = r\cos\theta$ and $y = r\sin\theta$

ii. The parametric equations of the circle with centre at (x_1, y_1) and radius r are $x = x_1 + r\cos\theta,\ y = y_1 + r\sin\theta$

7. Length of tangent from $p(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $PT = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$

The power of the point $p(x_1, y_1)$ w.r.t. a circle is $CP^2 - r^2$, where C is the centre and r is the radius of the circle.

8. If two circles touch each other externally, then the distance between their centres is equal to the sum of the radii (i.e. $C_1C_2=r_1+r_2$)

If two circles touch each other internally then, the distance between their centres is equal to difference of radii (i.e. $C_1C_2=r_1 \sim r_2$)

9. A line $y=mx+c$ is a tangent to any circle, then the distance between the centre and the line is equal to radius of the circle.

Condition for tangency: A line $y=mx+c$ is a tangent to the circle $x^2+y^2=a^2$ then $c^2=a^2\left(1+m^2\right)$

10. Radical axis: The radical axis of two circles is the locus of the point which moves such that the power of it w.r.t the circles are always equal.

Racial axis of

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and

$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is given by

$$2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$$

11. Definition of orthogonal circles:

Two circles are said to be orthogonal if the angle between them is a right angle.

Condition for two circles

$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ to intersect orthogonally is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

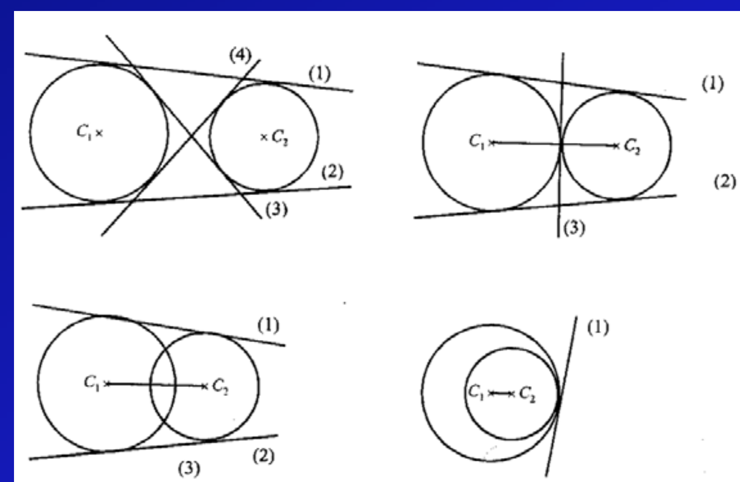
12. The numbers of common tangents to two circles

a. Will be four if $c_1c_2 > r_1 + r_2$

b. Will be three if $c_1c_2 = r_1 + r_2$

c. Will be two if $c_1c_2 < r_1 + r_2$

d. will be one if $c_1c_2 = r_1 \sim r_2$



13. Equation of tangent to the circle $x^2+y^2+2gx+2fy+c=0$ at (x_1, y_1) is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$

TIME SAVING RESULTS

1. If (x_1, y_1) is one end of the diameter of the circle $x^2+y^2+2gx+2fy+c=0$ Then the other end is $(-2g-x_1, -2f-y_1)$

2. i. Equation of the circle touching X-axis is of the form $x^2 + y^2 + 2gx + 2fy + g^2 = 0$

ii. Equation of the circle touching Y-axis is of the form $x^2 + y^2 + 2gx + 2fy + f^2 = 0$

iii. Equation of the circle touching both axes is of the form $x^2 + y^2 + 2gx + 2gy + g^2 = 0$

ii. Equation of the circle touching y-axis at (0, *b*) is of the form $x^2+y^2+2gx-2by+b^2=0$

4. The equation of the circle with centre at (*a, b*) and

i. touching X-axis is $x^2+y^2-2ax-2by+a^2=0$

ii. touching Y-axis is $x^2+y^2-2ax-2by+b^2=0$

5. Equation of the circle passing through the points (0, 0), (*a*, 0) and (0, *b*) is given by $x^2+y^2-ax-by=0$

6. The equation of the circumcircle of the triangle formed by the line $ax+by+c=0$ and the co-ordinate axes is

$$ab\left(x^2+y^2\right)+c(bx+ay)=0$$

7. Equation of the chord of the circle

i. $x^2+y^2=a^2$ bisected at (a, b) is given by $ax+by=a^2+b^2$

ii. $x^2+y^2+2gx+2fy+c=0$, bisected at (a, b) is given by $(a+g)x+(b+f)y=a(a+g)+b(b+f)$

8. The area of the triangle formed by the tangent at (x_1, y_1) on the circle with co-ordinate axes is $\dfrac{a^4}{2|x_1 y_1|}$.

9. If (x_1, y_1) is the mid point of the chord AB of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, then the length of the chord AB is

$$AB = \sqrt{-\left(x^2 + y^2 + 2gx_1 + 2fy_1 + c\right)}$$

10. Equation of any tangent to the circle

i. $x^2 + y^2 = a^2$ is given by $y = mx \pm a\sqrt{1+m^2}$

ii. $(x - x_1)^2 + (y - y_1)^2 = a^2$ is given by

$$y - y_1 = m(x - x_1) \pm a\sqrt{1+m^2}$$

QUESTIONS

1. The radius of the circle $9x^2+y^2=4(x^2-y^2)-8x$

1. $3/5$
2. $8/5$
3. $5/3$
4. $4/5$

Soln. The given circle can be written as

$$9x^2 + y^2 = 4x^2 - 4y^2 - 8x$$

$$\Rightarrow 5x^2 + 5y^2 + 8x = 0$$

$$\therefore 2g = \frac{8}{5} \Rightarrow g = \frac{4}{5}, f = 0, c = 0$$

$$\therefore \text{ Radius} = \sqrt{g^2 + f^2 - c} = \sqrt{\left(\frac{4}{5}\right)^2 + 0 - 0}$$

$$r = 4/5 \qquad \therefore \text{ Ans : (4)}$$

2. The radius of the circle $(x-y+a)^2+(y+x-a)^2=2a^2$ is

1. a
2. $\frac{a}{2}$
3. $2a^2$
4. $2a$

Soln. Given $(x-y+a)^2+(y+x-a)^2=2a^2$

$x^2+y^2+a^2-2xy-2ya+2ax+y^2+x^2+$

$a^2+2xy-2xa-2ay=2a^2$

$2x^2+2y^2-4ay=0$

$\div$ by 2 $\quad x^2+y^2-2ay=0$

Here $g=0, f=-a, c=0$

$\therefore r=\sqrt{g^2+f^2-c}=\sqrt{0+(-a)^2-0}$

$\therefore r=a \qquad \therefore$ Ans is (1)

3. The radius of the circle $(x-a)(x-b)+(y-c)(y-d)=0$ is

1. $\sqrt{(a-b)^2+(c-d)^2}$
2. $\sqrt{(a-b)^2-(c-d)^2}$
3. $(a-b)^2+(c-d)^2$
4. $\frac{1}{2}\sqrt{(a-b)^2+(c-d)^2}$

Soln. Given Circles is

$$(x-a)(x-b)+(y-c)(y-d)=0$$

$\therefore$ Ends of diameter are $A\equiv(a,\ c)$ and $B\equiv(b,d)$

$$\therefore \text{ Radius} = \frac{1}{2}AB$$

$$\therefore\ r=\frac{1}{2}\sqrt{(a-b)^2+(c-d)^2}$$

$\therefore$ Ans is (4)

4. The radius of the circle concentric with the circle $x^2 + y^2 - 6x + 12y + 15 = 0$ having 4 times its area is

1. $\sqrt{30}$
2. $3\sqrt{30}$
3. $2\sqrt{30}$
4. $4\sqrt{30}$

Soln.

Given circle is $x^2+y^2-6x+12y+15=0$

$$\begin{array}{l|l|l} 2g=-6 & 2f=12 & \\ & & c=15 \\ g=-3 & f=6 & \end{array}$$

$\therefore$ Area = πr^2 $\quad\therefore\ r=\sqrt{g^2+f^2-c}$

$=\pi(30)=4\times 30\pi$

$r=\sqrt{9+36-15}=\sqrt{30}$

$120\pi=\pi R^2 \Rightarrow R^2=120 \quad \therefore R=2\sqrt{30}$

$\therefore$ Ans. is (3)

5. The centre of the circle $x(y+x-6)=y(x-y+8)$ is

1. (4, 3)
2. (3, 4)
3. (–4, –3)
4. (–3, 4)

Soln. Given circle can be simplified as

$$xy + x^2 - 6x = xy - y^2 + 8y$$

$$\Rightarrow x^2 + y^2 - 6x - 8y = 0$$

$$\therefore 2g - 6 \quad 2f = -8 \quad c = 0$$

$$g = -3 \quad f = -4$$

$\therefore$ Centre is (3, 4)

$\therefore$ Ans (2)

6. The centre of the circle $ax^2+(2a-3)y^2-4x-1=0$ is

1. $(2, 0)$
2. $(-2/3, 0)$
3. $(2/3, 0)$
4. $(1/3, 0)$

Soln. Given circle equation is $ax^2+(2a-3)y^2-4x-1=0$

Here coefficient of x^2 = coefficient of y^2

$$a = 2a - 3 \Rightarrow a = 3$$

$$\therefore 3x^2 + 3y^2 - 4x - 1 = 0$$

$$\div \text{ by } 3\ ;\ x^2 + y^2 - \frac{4}{3}x - \frac{1}{3} = 0$$

$$\therefore 2g = -4/3 \qquad f = 0 \qquad c = -1/3$$

$$g = -2/3$$

$\therefore$ Centre is (2/3, 0) $\quad\therefore$ Ans is (3)

7. The centre and radius of the circle $x = 4 + 5\cos\theta$ and $y = 3 + 5\sin\theta$ is

1. (3, 5), 5
2. (4, 3), 5
3. (–3, 4), 5
4. None

Soln. Given $x = 4 + 5\cos\theta$ ....(1) and

$y = 3 + 5\sin\theta$ ......(2)

$x - 4 \equiv 5\cos\theta$ and $y - 3 = 5\sin\theta$

Squaring and adding

$(x-4)^2 + (y-3)^2 = 25$

$\therefore$ centre is (4, 3) and $r = 5$ $\therefore$ Ans is (2)

8. If one end of the diameter of the circle $2x^2 + 2y^2 - 4x - 8y + 2 = 0$ is (3, 2) then the other end is

1. (2, 3)
2. (4, –2)
3. (2, –1)
4. (–1, 2)

Soln. Given $2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\div$ by 2; $x^2 + y^2 - 2x - 4y + 1 = 0$

$$2g = -2 \quad 2f = -4 \quad c = 1$$

$$g = -1 \quad f = -2$$

$\therefore \quad c \equiv (1, 2)$ One end of diameter

$A \equiv (3,2)$

Other end $B(x, y) = ?$ Using mid – point

formula $\Rightarrow (1, 2) \equiv \left(\frac{x+3}{2}, \frac{y+2}{2} \right)$

$\Rightarrow (x,y) \equiv (-1, 2)$ $\quad \therefore$ Ans is (4)

9. The equation of the circle with centre (3, 1) and touching the line $8x - 15y + 25 = 0$ is

1. $x^2 + y^2 - 6x - 2y + 1 = 0$
2. $x^2 + y^2 - 6x - 2y + 3 = 0$
3. $x^2 + y^2 - 6x - 2y + 6 = 0$
4. None of these

Soln. Circle with centre (3, 1) is

$$(x-3)^2+(y-1)^2=a^2$$

This touches the line $8x-15y+25=0$,

then length of the $\perp$ from the centre = a

$$\frac{8.(3)-15.(1)+25}{\sqrt{64+225}}=a \Rightarrow \frac{34}{17}=a \;\therefore\; a=2$$

$\therefore$ circle is $(x-3)^2+(y-1)^2=2^2$

$$x^2+y^2-6x-2y+6=0$$

$\therefore$ Ans is (3)

10. The equation of a circle with centre at (1, 0) and circumference 10π units is

1. $x^2 + y^2 - 2x + 24 = 0$

2. $x^2 + y^2 - x - 25 = 0$

3. $x^2 + y^2 - 2x - 24 = 0$

4. $x^2 + y^2 + 2x + 24 = 0$

Soln. Given centre is (1, 0)

$c = 10\pi \qquad 2\pi r = 10\pi \Rightarrow r = 5$

$\therefore$ circle is $(x-1)^2 + (y-0)^2 = 5^2$

$x^2 + y^2 - 2x + 1 = 25$

$x^2 + y^2 - 2x - 24 = 0$

$\therefore$ Ans is (3)

11. The equation of the circle passing through (2, 1) and touching co-ordinate axes is

1. $x^2 + y^2 - 2x + 2y + 1 = 0$

2. $x^2 + y^2 + 2x - 2y + 1 = 0$

3. $x^2 + y^2 - 2x - 2y + 1 = 0$

4. $x^2 + y^2 + 2x + 2y - 1 = 0$

Soln. Circle touching both the axes is given by $x^2 + y^2 - 2ax - 2ay + a^2 = 0$

This passes through (2, 1)

$4+1-4a-2a+a^2 = 0$

$a^2 - 6a + 5 = 0 \Rightarrow a = 1, 5$

$\therefore$ one circle is $x^2 + y^2 - 2x - 2y + 1 = 0$

$\therefore$ Ans is (3)

12. The circles $x^2 + y^2 + 4x - 12y + 4 = 0$ and $x^2 + y^2 - 2x - 4y + 4 = 0$

1. Touch each other externally
2. Touch each other internally
3. Cut each other
4. Cut each other orthogonally

Soln. Centres of the circles are $c_1(-2,6)$ and $c_2(1,2)$.

Radii of the circles $r_1 = \sqrt{4+36-4} \Rightarrow r_1 = 6$

$$r_2 = \sqrt{1+4-4} \Rightarrow r_2 = 1$$

$$c_1c_2 = \sqrt{(1+2)^2 + (2-6)^2} \Rightarrow c_1c_2 = 5$$

$$\therefore\ c_1c_2 = 5 = r_1 - r_2$$

$\therefore$ given circles touch each other internally

$\therefore$ (2) is the correct answer

13. If the circles $x^2 + y^2 - 4x + 2y - 4 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ touch each other, then the point of contact is

1. (1, 7)
2. (–1, –7)
3. $\left(1/5, 7/5\right)$
4. $\left(7/5, 1/5\right)$

Soln. Centres of the circles are c_1 (2, –1) & c_2(–1, 3).

Radii of the circles

$$r_1 = \sqrt{4+1+4} \Rightarrow r_1 = 3$$

$$r_2 = \sqrt{1+9-6} \Rightarrow r_2 = 2$$

$$c_1c_2 = \sqrt{(2+1)^2 + (-1-3)^2},\ c_1c_2 = 5 = r_1 + r_2$$

Using Internal division formula

$$p \equiv \left(\frac{3.(-1)+2.2}{3+2}, \frac{3.3+2.(-1)}{3+2} \right) \Rightarrow p \equiv \left(\frac{1}{5}, \frac{7}{5} \right)$$

$\therefore$ (3) is the correct answer

14. If $2x^2 + axy + 2y^2 + (a-4)x + 6y - 5 = 0$ represents a circle then its area is

1. $23\pi/4$

2. $23\pi/2$

3. 23π

4. 46π

Soln. Given $2x^2+axy+2y^2+(a-4)x+6y-5=0$

is circle if $a=0$

$\therefore\ 2x^2+2y^2-4x+6y-5=0$

$$x^2+y^2-2x+3y-\frac{5}{2}=0$$

$\therefore g=-1,\ f=\frac{3}{2},\ c=\frac{-5}{2}\quad \therefore r=\sqrt{1+\frac{9}{4}+\frac{5}{2}}$

$\Rightarrow r=\sqrt{\frac{23}{4}}\ \therefore$ area $=\pi r^2=\frac{23\pi}{4}\ \therefore$ Ans (1)

15. The equations of the tangents to the circle $x^2 + y^2 = 12$ which makes an angle 60° with the X-axis is

1. $\sqrt{3}x \pm 2\sqrt{3} = y$

2. $\sqrt{2}x \pm 4\sqrt{3} = y$

3. $\sqrt{3}x \pm 4\sqrt{3} = y$

4. None

Soln. Circle equation is $x^2 + y^2 = 12$

Slope of tangent = m = tan60° $\Rightarrow m = \sqrt{3}$

Equation of tangent is $y = mx + c$

Where $m = \sqrt{3}$, $c^2 = a^2(1 + m^2)$

$$c^2 = 12(1 + 3) \Rightarrow c = \pm 4\sqrt{3}$$

$\therefore$ Equation of tangent is $y = \sqrt{3}x \pm 4\sqrt{3}$

$\therefore$ (3) is the correct answer

16. If $2y + x + 3 = 0$ touches the circle $5x^2 + 5y^2 = k$ then $k =$

1. 4
2. 9
3. 16
4. 25

Soln. Given circle is $5x^2 + 5y^2 = k$

$$\therefore\ x^2 + y^2 = \frac{k}{5} \qquad \therefore a^2 = \frac{k}{5}$$

$$\text{Line is} \quad 2y + x + 3 = 0, \quad 2y = -x - 3$$

$$y = -\frac{1}{2}.x = -\frac{3}{2}, (y = mx + c) \Rightarrow m = -\frac{1}{2}, \; c = \frac{-3}{2}$$

$$\therefore \text{ condition for tangency is } c^2 = a^2\left(1 + m^2\right)$$

$$\left(\frac{-3}{2}\right)^2 = \frac{k}{5}\left(1 + \left(\frac{-1}{2}\right)^2\right) \Rightarrow \frac{9}{4} = \frac{k}{5}.\frac{5}{4} \Rightarrow k = 9$$

$\therefore$ (2) is the correct answer

17. The length of the tangent drawn from (–2, 3) to the circle $2x^2 + 2y^2 = 3$ is

1. 5
2. 4
3. $\sqrt{\frac{23}{2}}$
4. $\frac{5}{\sqrt{2}}$

Soln. Given circle is $2x^2 + 2y^2 = 3$

$\therefore\ x^2 + y^2 = \dfrac{3}{2}$

$\therefore$ Length of tangent = $\sqrt{(-2)^2 + 3^2 - \dfrac{3}{2}}$

$= \sqrt{\dfrac{23}{2}}$

$\therefore$ (3) is the correct answer

18. The point (1, 2) lies inside the circle

1. $x^2 + y^2 + 2x - 4y + 4 = 0$

2. $x^2 + y^2 - 2x - 4y + 4 = 0$

3. $x^2 + y^2 + 2x + 4y - 4 = 0$

4. None

Soln. Substitute (1, 2) in all the 3 equations

i.e. $1+4+2-8+4=3 > 0$ lies outside ;

$1 + 4 - 2 - 8 + 4 = -1 < 0$

$\therefore$ Lies inside

$\therefore$ Required Answer is (2)

19. The number of tangents drawn to the circle $x^2 + y^2 - 8x - 6y + 9 = 0$ from the point (3, –2) is

1. 1
2. 2
3. 0
4. none

Soln. $S \equiv x^2 + y^2 - 8x - 6y + 9$

At (3, –2), $S \equiv 9 + 4 - 24 + 12 + 9$

$= 10 > 0$

$\therefore$ Point (3, –2) lies outside the circle

$\therefore$ two tangents can be drawn

$\therefore$ (2) is the correct answer

20. The radical axis of the circles

$3x^2+3y^2=4x-5y+1=0$ & $2x^2+2y^2=3x+2y-7$ is

1. $x-16y-23=0$
2. $x+23y-16=0$
3. $x+16y-23=0$
1. None

Soln. The circles can be written as

$$3x^2+3y^2-4x+5y-1=0 \Rightarrow x^2+y^2-\frac{4}{3}x+\frac{5}{3}y-\frac{1}{3}=0 \ldots 1$$

$$2x^2+2y^2-3x-2y+7=0 \Rightarrow x^2+y^2-\frac{3}{2}x-y+\frac{7}{2}=0.....2$$

$$\underline{-\quad -\quad +\quad +\quad -}$$

R.A. is (1) – (2) $\Rightarrow \left(\frac{3}{2}-\frac{4}{3}\right)x+\left(\frac{5}{3}+1\right)y-\left(\frac{1}{3}+\frac{7}{2}\right)=0$

$$\Rightarrow x+16y-23=0$$

$\therefore$ (3) is the correct answer

21. Which of the following is a point on common chord of circles $x^2+y^2+2x-3y+6=0$ and $x^2+y^2+x-8y-13=0$

1. (1, 2)
2. (1, –4)
3. (1, –2)
4. (1, 4)

Soln. Here the common chord of the given circles is the radical axis

$$\therefore x^2 + y^2 + 2x - 3y + 6 = 0$$

$$x^2 + y^2 + x - 8y - 13 = 0$$

$$- \quad - \quad - \quad + \quad +$$

R.A. is $x + 5y + 19 = 0$

The point (1, –4) lies on this line i.e.

1+5(–4)+19=0, 0 = 0

$\therefore$ (1, –4) lies on the common chord

$\therefore$ (2) is the correct answer

22. If the circles $x^2+y^2+2x+2ky+6=0$ and $x^2+y^2+2ky+k=0$ intersect orthogonally then k is

1. 2 or $-3/2$

2. –2 or $-3/2$

3. 2 or $3/2$

4. –2 or $3/2$

Soln. Given

$x^2+y^2+2x+2ky+6=0$ ($g_1=1, f_1=k, c_1=6$) &

$x^2+y^2+2ky+k=0$ ($g_2=0, f_2=k, c_2=k$)

Circles cut orthogonally if

$2g_1g_2+2f_2f_2=c_1+c_2 \Rightarrow 2.0+2k.k=6+k$

$2k^2-k-6=0 \Rightarrow k=2 \text{ or } -3/2$

$\therefore$ (1) is Answer

23. The circle $x^2 + y^2 - 6x - 8y + 9 = 0$ touches externally with a circle whose centre is origin. Then radius is equal to

1. 1
2. 16
3. 21
4. none

Soln. Given $x^2+y^2-6x-8y+9=0$ centre of 2nd is origin

$$2g=-6 \quad 2f=-8 \quad c=9 \qquad \text{i.e. } 0\,(0,0)$$

$$g=-3 \quad f=-4$$

$$\therefore\ c\equiv(3,4) \qquad \text{Now}$$

$$co=\sqrt{(3-0)^2+(4-0)^2}$$

$$r_1=\sqrt{9+16-9} \qquad co=\sqrt{9+16}\Rightarrow co=5$$

$$r_1=4 \qquad \text{Now } co=r_1+r_2,\ \ 5=4+r_2\Rightarrow r_2=1$$

$\therefore$ (1) is the correct answer

24. The tangent to $x^2 + y^2 - 2x - 3 = 0$ is parallel to X-axis at points

1. $(2, \pm\sqrt{3})$
2. $(1, \pm 2)$
3. $(\pm 1, 2)$
4. $(\pm 3, 0)$

Soln. Here we have to use calculus method for this example i.e. $x^2 + y^2 - 2x - 3 = 0,$

diff w.r.t. x $2x + 2y.\frac{dy}{dx} - 2 = 0 \Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$

Given tangent is parallel to X-axis

$\therefore$ slope$=\frac{dy}{dx}=m \Rightarrow m = 0$

$\therefore \frac{1-x}{y} = 0 \Rightarrow x = 1 \therefore y = \pm 2$

$\therefore$ (2) is the correct answer

25. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 sq.units. Then its equation is

1. $x^2 + y^2 + 2x - 2y - 62 = 0$

2. $x^2 + y^2 - 2x + 2y - 47 = 0$

3. $x^2 + y^2 - 12x - 2y - 47 = 0$

4. $x^2 + y^2 - 2x + 2y - 62 = 0$

Soln. Solving $2x-3y=5-x3 \Rightarrow 6x-9y=15$

$\therefore 2x-3(-1)=5$

$3x-4y=7-x2 \Rightarrow \underline{6x-8y=14}$ $\qquad 2x=2,\ x=1$

$-y=1 \Rightarrow y=-1$

$\therefore$ centre $\equiv (1, -1)$ given Area $= 154 \Rightarrow \pi r^2 = 154$

$$r^2 = \frac{154}{\pi} = \frac{154}{22} \times \frac{7}{1} \Rightarrow r^2 = 49,\ r = 7$$

$\therefore$ Equation of circle is $(x-1)^2+(y+1)^2=49$

$x^2-2x+1+y^2+2y+1-49=0$

$x^2+y^2-2x+2y-47=0 \therefore$ (2) is the correct Ans.

26. Equation of a circle passing through the points (1, 1), (5, –5) and (6, –4) is

1. $x^2 + y^2 + 6x - 4y = 0$

2. $x^2 + y^2 - 6x + 4y = 0$

3. $x^2 + y^2 - 6x + 4y - 10 = 0$

4. $x^2 + y^2 + 4x - 6y = 9$

Soln. Let the required equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Passes through (1, 1)

$\Rightarrow 1+1+2g+2f+c=0 \Rightarrow 2g+2f+c=-2$

Passes through (5,–5)

$\Rightarrow 25+25+\log-10f+c=0 \Rightarrow \log-10f+c=-50$

Passes through (6, –4)

$\Rightarrow 35+16+12g-8f+c=0 \Rightarrow 12g-8f+c=-52$

Solving these 3 we get $g = -3$, $f = 2$, $c = 0$

$\therefore$ Equation of circle is $x^2 + y^2 - 6x + 4y = 0$

$\therefore$ (1) is the correct answer **1**

Aliter : Putting (1, 1) in all the 4 equations the 2nd equation satisfies all the three points

27. If the points (0, 0), (1, 0), (0, 1) and (t, t) are concyclic then t is equal to

1. –1
2. 1
3. 2
4. –2

Soln. Let required equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It passes through (0, 0) $\Rightarrow c = 0$

Passes through (1,0)$\Rightarrow$1+2g=0$\Rightarrow$g=–1/2

Passes through (0,1)$\Rightarrow$1+2*f*=0$\Rightarrow$*f*=–1/2

$\therefore$ Equation of circle is $x^2 + y^2 - x - y = 0$

for concyclic (*t, t*) must lie on this circle

i.e. $t^2 + t^2 - t - t = 0 \Rightarrow 2t^2 - 2t = 0$

$2t(t-1) = 0 \Rightarrow t = 0$ or 1

$\therefore$ (2) is correct answer

28. Equation of circle through origin, has centre on $x+y-4=0$ and cuts $x^2+y^2-4x+2y+4=0$ orthogonally is

1. $x^2+y^2-2x-6y=0$
2. $x^2+y^2-4x-4y=0$
3. $x^2+y^2-6x-3y=0$
4. $x^2+y^2+4x-2y=0$

Soln. Let required equation of circle be

$x^2+y^2+2gx+2fy+c=0$

Passes through origin $\therefore c=0 \Rightarrow x^2+y^2+2gx+2fy=0$

Centre lies on $x+y-4=0 \Rightarrow -g-f-4=0$ …(1)

Cuts orthogonally the circle $x^2+y^2-4x+2y+4=0$

$-4g+2f=4 \Rightarrow -2g+f=2$ …(2)

Solving (1) & (2)

$$-g-f=4$$
$$-2g+f=2$$
$$-3g=6 \Rightarrow g=-2$$

$$-(-2)-f=4$$
$$f=-2$$

$\therefore$ Equation of circle is $x^2+y^2-4x-4y=0$

1

$\therefore$ (2) is correct answer

29. A circle touches the Y-axis at (0, 2) and its X-intercept is 3 then equation of the circle is

1. $x^2 + y^2 \pm 5y + 4 = 0$
2. $x^2 + y^2 \pm 5x - 4y + 4 = 0$
3. $x^2 + y^2 + 5x \pm 4y + 4 = 0$
4. $-x^2 + y^2 \pm 5x + 4y - 4 = 0$

Soln. Let the radius of required circle be a'

Centre $\equiv (a, 2)$

$\therefore$ Equation is $(x-a)^2+(y-2)^2=a^2$

$x^2-2ax+a^2+y^2-4y+4=a^2$

$x^2+y^2-2ax-4y+4=0$

To find a' use X-intercept

i.e. $2\sqrt{g^2-c}=3 \Rightarrow 2\sqrt{a^2-4}=3,$

$\sqrt{a^2-4}=3/2 \Rightarrow a=\pm 5/2$

$\therefore$ Equation of circle is $x^2+y^2\pm 5x-4y+4=0$

$\therefore$ (2) is the answer

30. If a circle $x^2 + y^2 - 17x + 2fy + c = 0$ passes through (3, 1) (14, –1) and (11, 5) then c =

1. 0
2. –41
3. –17/2
4. 41

Soln. Substitute (3, 1) in given circle is

$9+1-51+2f+c=0 \Rightarrow 2f+c=41$

Substitute (14, –1) in given circle

i.e. $14^2+1-17(14)+2f(-1)+c=0$

$\Rightarrow -2f+c=41$

$\therefore c = 41$

$\therefore$ (4) is the answer

31. Two circles of equal radius *r* cut orthogonally if their centres are (2, 3) and (5, 6) then r =

1. 1
2. 2
3. 3
4. 4

Soln. $(x-2)^2+(y-3)^2=r^2$

$$\Rightarrow x^2+y^2-4x-6y+13-r^2=0$$

$(x-5)^2+(y-6)^2=r^2$

$$\Rightarrow x^2+y^2-10x-12y+61-r^2=0$$

Using $2g_1g_2+2f_1f_2=c_1+c_2$

$$-4(-5)+(-6)(-6)=13-r^2+61-r^2$$

$2r^2=74-56\Rightarrow 2r^2=18,\ r^2=9\Rightarrow r=3$

$\therefore$ (3) is the correct answer

32. The line $y = x$ is tangent at (0, 0) to a circle of radius unity. The centre of the circle is

1. (1, 0)
2. $\left(-1/\sqrt{2}, 1/\sqrt{2}\right)$
3. $\left(1/\sqrt{2}, -1/\sqrt{2}\right)$
4. $\left(-1/\sqrt{2}, -\frac{1}{\sqrt{2}}\right)$

Soln. Given r=1 Let $c \equiv (h, k)$

$\therefore 1 = \sqrt{(h-0)^2 + (k-0)^2} \Rightarrow h^2 + k^2 = 1 \ldots(1)$

Length of $\perp$ from centre to the line $y = x$

$$= r \Rightarrow \left| \frac{1.(h) - 1(k) + 0}{\sqrt{1+1}} \right| = 1$$

$$\Rightarrow h - k = \sqrt{2} \Rightarrow h = k + \sqrt{2} \ldots.(2)$$

Substitute (2) in (1) $\left(k + \sqrt{2}\right)^2 + k^2 = 1$

$$k^2 + 2\sqrt{2}k + 2 + k^2 - 1 = 0 \Rightarrow$$
$$2k^2 + 2\sqrt{2}k + 1 = 0$$
$$\Rightarrow \left(\sqrt{2}k + 1\right)^2 = 0 \Rightarrow k = \frac{-1}{\sqrt{2}}$$
$$\therefore\ h = -\frac{1}{\sqrt{2}} + \sqrt{2} \Rightarrow h = \frac{1}{\sqrt{2}}$$
$$\therefore \text{centre is} \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$
$\therefore$(3) is the correct Ans

33. The gradient of tangent at (6, 8) to $x^2 + y^2 = 100$ is

1. $3/4$
2. $4/3$
3. $-3/4$
4. $-4/3$

Soln. Given $C \equiv (0, 0)$, $P = (6, 8)$

$\therefore$ Slope of radius = $\dfrac{8-0}{6-0} = \dfrac{4}{3}$

$\therefore$ gradient of tangent = $\dfrac{-3}{4}$

$(\because \text{tangent} \perp r)$

$\therefore$(3) is the correct answer

34. Any point on circle $x^2+y^2+4x-2y=0$ is given by

1. $\left(\sqrt{5}\cos\theta, \sqrt{5}\sin\theta\right)$

2. $\left(\sqrt{5}\sin\theta, \sqrt{5}\cos\theta\right)$

3. $\left(-2+\sqrt{5}\cos\theta, 1+\sqrt{5}\sin\theta\right)$

4. $(\cos\theta, \sin\theta)$

Soln. Substitute all the 4 points. The point $\left(-2+\sqrt{5}\cos\theta, +\sqrt{5}\sin\theta\right)$

i.e. LHS = $\left(-2+\sqrt{5}\cos\theta\right)^2+\left(1+\sqrt{5}\sin\theta\right)^2+$ $4\left(-2+\sqrt{5}\cos\theta\right)-2\left(1+\sqrt{5}\sin\theta\right)$

$=4-4\sqrt{5}\cos\theta+5\cos^2\theta+1+2\sqrt{5}\sin\theta+$ $5\sin^2\theta-8+4\sqrt{5}\cos\theta-2-2\sqrt{5}\sin\theta$

$=5+5\left(\cos^2\theta+\sin^2\theta\right)-10=0$

$\therefore$ (3) is the correct answer

35. The shortest distance between (0, 5) to circumference of circle $x^2+y^2-10x+14y-151=0$ is

1. 13
2. 9
3. 3
4. 5

soln. Centre $\equiv c \equiv (5, -7)$ and $A \equiv (0, 5)$

$r = \sqrt{25 + 49 + 151} \Rightarrow r = \sqrt{225},\ r = 15$

$CA = \sqrt{5^2 + 12^2} = CA = 13$

$QA = |CA - r| \Rightarrow P = |15 - 13|,\ PA = 2$

$\therefore$ (3) is the correct answer

36. Length of the chord and circle $x^2 + y^2 + 3x + 2y - 8 = 0$ intercepted by y-axis is

1. 3
2. 8
3. 9
4. 6

Soln. Putting $x = 0$ in circle equation

$y^2 + 2y - 8 = 0 \Rightarrow$ solving

$y = 2, -4$

$\Rightarrow$ length is 6

$\therefore$ (4) is answer

Aliter : Length of y – intercept $= 2\sqrt{f^2 - c}$

$= 2\sqrt{1 + 8} = 2.3 = 6$

37. Two lines $3x - 2y - 8 = 0$ and $2x - y - 5 = 0$ are two diameters and circle touches X-axis then equation of circle

1. $(x-2)^2 + (y-1)^2 = 1$
2. $(x+2)^2 + (y-1)^2 = 1$
3. $(x-2)^2 + (y+1)^2 = 1$
4. $(x+2)^2 + (y+1)^2 = 1$

Soln. Given 3x–2y=8..(1)   2x–y=5…(2)

(1) – (2)×2

$$3x-2y=8 \qquad\qquad y=2x-5$$

$$4x-2y=10 \qquad\qquad y=2.2-5$$

$$\underline{-\quad +\qquad\quad -}$$

$$-x=-2 \qquad x=2 \qquad\qquad y=-1$$

$$\therefore c\equiv(2,-1) \qquad \therefore r=1$$

$\therefore$ Equation of circle is $(x-2)^2+(y+1)^2=1$

$\therefore$ (3) is the correct answer

38. The equation of tangent to $x^2 + y^2 = 25$, which makes 45^0 angle with X-axis

1. $x = y$

2. $y = x \pm 5\sqrt{2}$

3. $x = y \pm 5\sqrt{2}$

4. $x + y = 5\sqrt{2}$

Soln. Let equation of tangent be $y=mx+c$

$m = \tan 45^0$ using $c^2 = a^2(1+m^2)$

$m = 1$ $c^2 = 25(1+1) \Rightarrow c = \pm 5\sqrt{2}$

$\therefore$ Equation is $y = x \pm 5\sqrt{2}$

$\therefore$ (2) is the correct answer

39. Length of the chord of circle $x^2+y^2=9$ intercepted by the line $x+2y-3=0$ is

1. 8
2. $12\sqrt{5}$
3. $12/\sqrt{5}$
4. $5\sqrt{12}$

Soln. Given $x+2y-3=0$ putting in $x^2+y^2=9$

$\Rightarrow x=3-2y$ $\quad (3-2y)^2+y^2=9$

$9+4y^2-12y+y^2=0$

$\Rightarrow 5y^2-12y=0 \Rightarrow y=0$ or $y=12/5$

If $y = 0$ $\quad \therefore x=3 \quad \therefore A\equiv(3, 0)$

$y = 12/5 \quad \therefore x=3-12/5 \quad \therefore (-9/5, 12/5)=B$

$x=-9/5$

$$\therefore AB = \sqrt{\left(3+\frac{9}{5}\right)^2 + \left(0-\frac{12}{5}\right)^2}$$

$$= \sqrt{\left(\frac{24}{5}\right)^2 + \left(\frac{12}{5}\right)^2} = \sqrt{\frac{576+144}{25}} = \sqrt{\frac{720}{25}}$$

$$AB = \sqrt{\frac{144}{5}} \Rightarrow AB = \frac{12}{\sqrt{5}}$$

$\therefore$ (3) is the correct answer

40. The length of the tangent from (–1, 2) to the circle $4x^2 + 4y^2 - 8x + y - 6 = 0$ is

1) 24

2) $\sqrt{24}$

3) 76

4) $\sqrt{6}$

Soln. Given $4x^2 + 4y^2 - 8x + y - 6 = 0$

$\div$ by 4; $x^2 + y^2 - 2x + \frac{1}{4}y - \frac{3}{2} = 0$

$\therefore$ length of tangent from (–1, 2)

$$= \sqrt{1 + 4 - 2(-1) + \frac{1}{4}(2) - \frac{3}{2}}$$

$$= \sqrt{\frac{10 + 4 + 1 - 3}{2}} = \sqrt{6}$$

$\therefore$(4) is the correct answer

41. The angle between a pair of tangents from a point P to circle $x^2+y^2+4x-6y+9\sin^2\alpha+13\cos^2\alpha=0$ is 2α. The equation of the locus of P is

1. $x^2+y^2+4x-5y+4=0$
2. $x^2+y^2+4x-6y-9=0$
3. $x^2+y^2+4x-6y-4=0$
4. $x^2+y^2+4x-6y+9=0$

Soln. Centre is $(-2, 3)$

$$r = \sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha}$$

$$= \sqrt{4 + 9\left(1 - \sin^2\alpha\right) - 13\cos^2\alpha}$$

$$= \sqrt{4 + 9\cos^2\alpha} = 2\sin\alpha = AC$$

$$\therefore \ \sin\alpha = \frac{AC}{PC} = \frac{2\sin\alpha}{\sqrt{(h+2)^2 + (k-3)^2}}$$

$$\Rightarrow (h+2)^2+(k-3)^2=4 \Rightarrow h^2+4h+4+k^2-6k+9=4$$

Simplifying & putting $h = x,\ k = y$

$$x^2 + y^2 + 4x - 6x + 9 = 0$$

$\therefore$ (4) is the correct answer

42. Circumcentre of the triangle whose vertices are (0, 0), (6, 0) and (0, 4) is

1. (2, 0)
2. (3, 0)
3. (0, 3)
4. (3, 2)

Soln. Let required equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

It passes through (0, 0) $\therefore$ $c = 0$

It passes through (6,0)$\Rightarrow$36+12g=0$\Rightarrow$$g$=–3

It passes through (0,4)$\Rightarrow$16+8f=0$\Rightarrow$$f$=–2

$\therefore$ Equation of circle is $x^2+y^2-6x-4y=0$

$\therefore$ (4) is the correct answer

43. The value of k such that the equation $2x^2 + 2y^2 - 6x + 8y + k = 0$, represents a point circle is

1. 0
2. 25
3. $\frac{25}{2}$
4. $\frac{-25}{2}$

Soln. Given $2x^2 + 2y^2 - 6x + 8y + k = 0$

$\div$ by 2; $x^2 + y^2 - 3x + 4y + \frac{k}{2} = 0 \quad \therefore g = \frac{-3}{2},$

$f = 2, \qquad c = \frac{k}{2}$ for point circle

$$r = 0 \Rightarrow \sqrt{\left(\frac{-3}{2}\right)^2 + 4 - \frac{k}{2}} = 0,\ \frac{9}{4} + 4 - \frac{k}{2} = 0$$

$$\Rightarrow \frac{25}{4} = \frac{k}{2} \Rightarrow k = \frac{25}{2}$$ $\therefore$ (3) is the correct Ans

44. If (x, y) and (3, 5) are the extremities of a diameter of a circle with centre at (2, 3) then the values of x and y are

1. $x = 1, y = 4$

2. $x = 4, y = 1$

3. $x = 8, y = 2$

4. none

Soln. By mid point formula

$$(2, 3) \equiv \left(\frac{x+3}{2}, \frac{y+5}{2} \right)$$

$$\Rightarrow x+3=4 \Rightarrow x=1 \qquad y+5=6 \Rightarrow y=1$$

$$\therefore x=y=1$$

$\therefore$(4) is the answer

45. If two circles $a\left(x^2+y^2\right)+bx+cy=0$ and $A\left(x^2+y^2\right)+Bx+Cy=0$ touch each other then

1. $aC=cA$
2. $bC=cB$
3. $aB=bA$
4. $aA=bB=cC$

$\therefore$ Equation of tangent at (0, 0) is

$$x.0 + y.0 + \frac{b}{2a}(x+0) + \frac{c}{2a}(y+0) = 0 \text{ and}$$

$$x.0 + y.0 + \frac{B}{2A}(x+0) + \frac{C}{2A}(y+0) = 0$$

i.e. $bx + cy = 0$ and $Bx + Cy = 0$. These two are identical

$$\therefore \frac{b}{B} = \frac{c}{C} \Rightarrow bC = cB \therefore (2) \text{ is the correct Ans}$$

46. Y-axis is a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ if

1. $g^2 < c$

2. $f^2 < c$

3. $g^2 = c$

4. $f^2 = c$

Soln. Y-axis a tangent to given circle if

$$r = |g|$$

$$\sqrt{g^2 + f^2 - c} = g \Rightarrow g^2 + f^2 - c = g^2$$

$$\Rightarrow f^2 = c$$

$\therefore$ (4) is the correct answer

47. The value of k for which the circles $x^2+y^2-3x+ky-5=0$ and $4x^2+4y^2-12x-y-9=0$ becomes concentric is

1. $1/8$

2. $-1/8$

3. $1/4$

4. $-1/4$

Soln. Given

$$x^2+y^2-3x+ky-5=0 \qquad 4x^2+4y^2-12x-y-9=0$$

$$g=-3/2,\ f=k/2 \qquad \div \text{by}4;\ x^2+y^2-3x-\frac{1}{4}y-\frac{9}{4}=0$$

$$\therefore c_1=(+3/2,-k/2) \qquad g_2=-3/2,\ f_2=-1/8$$

$$\therefore c_1=c_2 \qquad \therefore c_2\equiv(3/2,1/8)$$

$$\frac{-k}{2}=\frac{1}{8}\Rightarrow k=-\frac{1}{4}$$

$\therefore$(4) is the correct answer

48. The equation of the chord of the circle $x^2 + y^2 - 4x = 0$, whose mid – point is (1, 0) is

1. $y = 2$

2. $y = 1$

3. $x = 2$

4. $x = 1$

Soln. Centre of given circle is $C(2, 0)$

Mid point of chord is $A(1, 0)$

Slope of $CA = \dfrac{0-0}{2-1} = 0$

Slope of $PQ = \dfrac{-1}{0}$

$\therefore$ Equation of PQ is $y - 0 = \dfrac{-1}{0}(x-1) \Rightarrow x = 1$

$\therefore$ (4) is the correct answer

49. Which of the following lines is a normal to the circle $(x-1)^2+(y-2)^2=10$

1. $x+y=3$

2. $(x-1)+(y-2)=10$

3. $x+2y=10$

4. $2x+y=3$

Soln.

Centre of the given circle is (1, 2)

Verify which line satisfies this point.

Clearly (1, 2) lies on $x + y = 3$

$\therefore$ (1) is the correct answer

50. The number of common tangents to the circles $x^2+y^2-x=0$ and $x^2+y^2+x=0$ is

1. 2
2. 1
3. 4
4. 3

Soln. Given $x^2+y^2-x=0$ $\quad x^2+y^2+x=0$

$$g=\frac{-1}{2}, f_1=0, c_1=0 \quad g_2=\frac{1}{2}, f_2=0, c_2=0$$

$C_1 \equiv (1/2, 0) \qquad C_2 = [-1/2, 0]$

$r_1 = \sqrt{(1/2)^2} = 1/2$

$r_2 = \sqrt{(-1/2)^2} = 1/2$

$\therefore C_1C_2 = \sqrt{\left(\frac{1}{2} + \frac{1}{2}\right)^2 + 0} = 1$

$\therefore r_1 + r_2 = C_1C_2 \Rightarrow$ circles touch externally

$\therefore$ 3 tangents can be drawn

$\therefore$ (4) is the correct answer

51. The equation of the circle with (3, 4) and (4, 3) as ends of a diameter is

1. $x^2 + y^2 + 7x + 7y + 24 = 0$

2. $x^2 + y^2 - 6x - 8y + 25 = 0$

3. $x^2 + y^2 - 7x - 7y + 24 = 0$

4. None

Soln. Equation of circle is given by

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$$(x - 3)(x - 4) + (y - 4)(y - 3) = 0$$

$$x^2 - 7x + 12 + y^2 - 7y + 12 = 0$$

$$x^2 + y^2 - 7x - 7y + 24 = 0$$

$\therefore$ (3) is the correct answer

52. The equation of the circle passing through the point (–7, 1) having centre (–4, –3) is

1. $x^2 + y^2 + 8x + 6y = 0$

2. $x^2 + y^2 + 4x + 3y = 0$

3. $x^2 + y^2 - 8x - 6y = 0$

4. None

Soln Given $C \equiv (-4, -3)$ & $P \equiv (-7, 1)$

$CP = r \Rightarrow (-4+7)^2 + (-3, -1)^2 = r^2$

$\Rightarrow r^2 = 9 + 16 \Rightarrow r = 5$

$\therefore$ Equation of circle is

$(x+4)^2 + (y+3)^2 = 25$

$\Rightarrow x^2 + y^2 + 8x + 6y = 0$

$\therefore$ (1) is the correct answer

53. Equation of circle through (2, 0), having centre on $3x - y - 5 = 0$ and length of tangent from (3, 1) is $\sqrt{8}$ is

1. $x^2 + y^2 - 2x + 4y = 0$
2. $x^2 + y^2 - 2x - 8y = 0$
3. $x^2 + y^2 + 4y - 3x + 1 = 0$
4. $x^2 + y^2 + 2x - 8y - 6 = 0$ or none

Soln. Let the required equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It passes through (2, 0) i.e. $4+4g+c=0$

$$\Rightarrow 4g+c=-4 \quad \ldots.(1)$$

Centre on $3x-y=5$ i.e. $-3g+f=5 \quad \ldots.(2)$

Length of tangent from (3, 1) is $\sqrt{8}$

$\sqrt{8}=\sqrt{9+1+6g+2f+c} \Rightarrow 6g+2f+c=-2 ..(3)$

$$4g+c=-4$$

(1)–(3) $6g+2f+c=-2$

$$\underline{-\quad -\quad -\quad +}$$

$$-2g-2f=-2$$

$$c=-4-4\left(-\frac{3}{4}\right)$$

$$c=-1$$

$$-3\left(-3/4\right)+f=5$$

$\div$ by(2)

$$-g-f=-2$$
$$-3g+f=5$$
$$\overline{-4g=3}$$
$$g=\frac{-3}{4}$$

$$f=5-\frac{9}{4}=\frac{11}{4}$$

$$4\left(\frac{-3}{4}\right)+c=-4$$

$$\therefore\ x^2+y^2+2\left(\frac{-3}{4}\right)x+2.\frac{11}{4}y-1=0$$

$\therefore$ Answer is (4)

54. If the square of the lengths of the tangents from P to $x^2 + y^2 = b^2$ and $x^2 + y^2 = c$ are in A.P. Then $a^2,\ b^2,\ c^2$ are in

1. AP
2. GP
3. HP
4. AGP

Soln. Given $PT_1^2 = x^2 + y^2 - a^2$,

$PT_2^2 = x^2 + y^2 - b^2$, $PT_3^2 = x^2 + y^2 - c^2$

$PT_2^2 - PT_1^2 = PT_3^2 - PT_2^2$

$x^2 + y^2 - b^2 - x^2 - y^2 + a^2$

$= x^2 + y^2 - c^2 - x^2 - y^2 + b^2$

$-b^2 + a^2 = -c^2 + b^2$

$\Rightarrow c^2 - b^2 = b^2 - a^2 \Rightarrow a^2, b^2, c^2$ are in A.P

$\therefore$ (1) is the correct answer

55. Area of triangle formed by +ve X-axis, tangent and normal to $x^2 + y^2 = 4$ at $\left(1, \sqrt{3}\right)$ is

1. $2\sqrt{3}$
2. $4\sqrt{3}$
3. $3\sqrt{3}$
4. $\sqrt{3}$

Soln. Tangent at $P\left(1,\sqrt{3}\right)$ IS

$x.1+y.\sqrt{3}=4$ using $xx_1+yy_1=a^2$

Put $y=0$ in this $\therefore x=4$

$\therefore A\equiv\left(4,0\right)$

$\therefore \Delta POA=\frac{1}{2}PM.OA \quad =\frac{1}{2}.\sqrt{3}.4$ here OP is normal

$=2\sqrt{3}$

$\therefore$ (1) is the correct answer

56. The lines $3x–4y+4=0$ and $6x–8y–7=0$ are tangent to the same circle. The radius of the circle is

1. $4/5$
2. $7/10$
3. $3/4$
4. $3/2$

Soln. Any point on the line $3x - 4y + 4 = 0$ is (0, 1). Distance from (0, 1) to $6x - 8y - 7 = 0$

$$= \left|\frac{0 - 8.1 - 7}{\sqrt{36 + 64}}\right| \Rightarrow \frac{15}{10} = \frac{3}{2}$$

= diameter of circle

$\therefore$ Radius = $\frac{3}{4}$

$\therefore$ (3) is the correct answer

57. The slope of the normal to the circle $x^2 + y^2 - 16x + 12y + 75 = 0$ at the point (5, –2) is

1. $4/5$
2. $3/4$
3. $-4/3$
4. $-3/4$

Soln. Equation of tangent at (5, –2)

$$x.5 + y(-2) - 16.5 + 12(-2) + 75 = 0$$

$$\Rightarrow -3x + 4y + 23 = 0$$

$\therefore$ Slope of tangent = 3 / 4

Slope of normal = $-4/3$

$\therefore$ (3) is the answer

58. If $3x - 4y + k = 0$, is a tangent to the circle $(x-1)^2 + (y-1)^2 = 2^2$ then $k =$

1. ± 10

2. 9, –11

3. 11

4. None

Soln. Given circle is $(x-1)^2+(y-1)^2=2^2$

$\therefore\ c \equiv (1,1),\ r=2$

The line $3x-4y+k=0$ is a tangent to circle if

$$r=\left|\frac{3.1+(-4)1+k}{\sqrt{9+16}}\right| \Rightarrow 2=\pm\left(\frac{3-4+k}{5}\right)$$

$\Rightarrow 10=\pm(-1+k) \Rightarrow 10=-1+k$ or

$10=-(-1+k),\ k=11$ or $-k=9 \Rightarrow k=-9$

$\therefore$ (4) is the correct answer

59. The equation $x^2+y^2+4x+6y+13=0$ represents

1. Circle
2. A pair of two distinct lines
3. A pair of coincidental lines
4. A point circle

Soln. Given $x^2 + y^2 + 4x + 6y + 13 = 0$

$g = 2, f = 3, c = 13$

$\therefore r = \sqrt{4 + 9 - 13} \Rightarrow r = 0$

$\therefore$ A point circle

$\therefore$ (4) is the correct answer

60. If $y = x + a\sqrt{2}$ touches the circle $x^2 + y^2 = a^2$ at the point

1) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$

2) $\left(\frac{-a}{\sqrt{2}}, \frac{-a}{\sqrt{2}}\right)$

3) $\left(\frac{a}{\sqrt{2}}, \frac{-a}{\sqrt{2}}\right)$

4) $\left(\frac{-a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$

Soln. Let the lint $y = x + a\sqrt{2}$ touch the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1).

Tangent at (x_1, y_1) is $xx_1 + yy_1 - a^2 = 0$

$x - y + a\sqrt{2} = 0$ is also a tangent

$\therefore \dfrac{x_1}{1} = \dfrac{y_1}{-1} = \dfrac{-a^2}{a\sqrt{2}} \qquad x_1 = \dfrac{-a}{\sqrt{2}}, y_1 = \dfrac{a}{\sqrt{2}}$

$\therefore$ point of contact is $\left(\dfrac{-a}{\sqrt{2}}, \dfrac{a}{\sqrt{2}}\right)$

$\therefore$ (4) is the correct answer

61. The radius of the circle with centre at (0, 2) and cutting orthogonally to the circle $x^2 + y^2 - 2x + y = 0$ is

1. 1
2. 2
3. $\sqrt{2}$
4. $\sqrt{6}$

Soln. Given circle is $x^2 + y^2 - 2x + y = 0$

Equation of circle with centre at (0, 2) is

$x^2 + y^2 - 4y + c = 0$ these two cut

orthogonally $2.(-1).0 + 2.\frac{1}{2}.(-2) = c \Rightarrow c = -2$

$\therefore\ x^2 + y^2 - 4y - 2 = 0$

$\therefore r = \sqrt{4+2} \Rightarrow r = \sqrt{6}$

$\therefore$ (4) is the correct answer

62. If $2x - 3y = 0$ is the equation of the common chord of the circles $x^2 + y^2 + 4x = 0$ and $x^2 + y^2 + 2\lambda y = 0$, then $\lambda =$

1. 3
2. 2
3. 1
4. 0

Soln. Common chord is given by

$$x^2 + y^2 + 4x = 0$$

$$x^2 + y^2 + 2\lambda y = 0$$

$$- \quad - \quad -$$

$$4x - 2\lambda y = 0 \quad \Rightarrow 2x - \lambda y = 0$$

Compare with $2x - 3y = 0$ $\lambda = 3$

$\therefore$ (1) is the correct answer

63. The abscissa of two points A and B are the roots of $x^2 - 4x + 2 = 0$ and their ordinates are the roots of $x^2 + 6x - 3 = 0$. The equation of the circle with AB as diameter is

1. $x^2 + y^2 + 4x - 6y + 1 = 0$
2. $x^2 + y^2 - 4x - 6y + 1 = 0$
3. $x^2 + y^2 - 4x + 6y - 1 = 0$
4. $x^2 + y^2 + 4x + 6y + 1 = 0$

Soln. Let x_1 and x_2 be the roots of $x^2-4x+2=0$ and y_1 and y_2 are the roots of $x^2+6x-3=0$.

$[x_1+x_2=4,\ x_1x_2=2,\ y_1+y_2=-6, yy_2=-3]$

The equation of the circle with (x_1, y_1) and (x_2, y_2) as the ends of the diameter is

$x^2+y^2-(x_1+x_2)x-(y_1+y_2)y+(x_1x_2+y_1y_2)=0$

$x^2+y^2-(4)x-(-6)y+(2-3)=0$

$x^2+y^2-4x+6y-1=0$ $\therefore$ (3) is the Ans.

64. If a circle passes through the point (a,b) and cuts the circle $x^2 + y^2 = 4$, orthogonally, then the locus of its centre is

1. $2ax - 2by + (a^2 + b^2 + 4) = 0$
2. $2ax + 2by - (a^2 + b^2 + 4) = 0$
3. $2ax + 2by + (a^2 + b^2 + 4) = 0$
4. $2ax - 2by - (a^2 + b^2 + 4) = 0$

Soln. Let the required circle be $x^2+y^2+2gx+2fy+c=0$.

This cuts orthogonally $x^2+y^2-4=0$

$$2g_1.0+2f_1.0=c-4 \Rightarrow c=4$$

req. also passes through (a, b)

$$a^2+b^2+2ag+2bf+4=0$$

$\therefore$ locus of the centre $(-g,-f)$ is

$$2ax+2by-\left(a^2+b^2+4\right)=0$$

$\therefore$ (2) is the correct answer

65. The radius of the circle $x^2 + y^2 - 2x\cos t + 2y\sin t - 15 = 0$ is

1. 3
2. 4
3. 5
4. $\sin t \cos t$

Soln.

Given $x^2 + y^2 - 2x\cos t + 2y\sin t - 15 = 0$

$g = -\cos t,\ f = \sin t,\ c = -15$

$r = \sqrt{\cos^2 t + \sin^2 t + 15} \Rightarrow r = \sqrt{16}$

$\therefore$(2) is the correct answer

66. The area of a circle with centre at (1,2) and passing through (4, 6) is

1. 30π
2. 5π
3. 15π
4. 25π

Soln. Given $c \equiv (1, 2)$

Circle passes through $p(4,6)$ then $CP = r$

$$\sqrt{(4-1)^2 + (6-2)^2} = r \Rightarrow r = 5$$

$\therefore$ Area of circle = $\pi r^2 = 25\pi$

$\therefore$(4) is the correct answer

67. The equations $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta, 0 \le \theta \le 2\pi$ represents a

1. a circle
2. degenerate circle
3. an empty set
4. a pair of st. lines

Soln. Given $x = a\cos\theta + b\sin\theta$..... (1)

$y = a\sin\theta - b\cos\theta$ .....(2)

Squaring and adding

$x^2 + y^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos$

$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$

Simplify $x^2 + y^2 = a^2 + b^2$

$\therefore$ (1) is the correct answer

68. $x^2 + y^2 - 7x + 8y - 11 = 0$ is a circle. The points (0, 0) and (1, 8) lie

1. both inside the circle
2. both outside the circle
3. one outside the circle & one inside
4. one on the circle and the other outside

Soln. Given $x^2 + y^2 - 6x + 8y - 11 = 0$

$PT_1^2 = 0 + 0 - 0 + 0 - 11 = -11 < 0$

i.e. (0, 0) lies inside the circle

$PT_2^2 = 1 + 64 - 6 + 64 - 11$= 112>0

i.e. (1, 8) lies on the circle

$\therefore$ (3) is the correct answer

69. The centres of the circles $x^2 + y^2 = 1$, $x^2 + y^2 + 6x - 2y - 1 = 0$ and $x^2 + y^2 - 12x + 4y - 1 = 0$ lie on

1. a circle
2. a st. line
3. $x^2 = 9y$
4. None

Soln Let centres of these circles be A, B & C

$A \equiv (0, 0), \quad B \equiv (-3, 1), \; C \equiv (6, -2)$

$\therefore \; AB = \sqrt{9+1} = \sqrt{10}$

$\therefore BA + AC = BC$

$BC = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$

$\therefore$ centres lie

$CA = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

on a St. line

1

$\therefore$ (2) is the correct answer

70. The square of the length of the tangent from (3, –4) to the circle $x^2 – y^2 – 4x – 8y + 3 = 0$ is

1. 20
2. 30
3. 48
4. 50

Soln.

$$PT^2 = x_1^2 + y_1^2 - 4x_1 - 8y_1 + 3,$$

put $x_1=3$, $y_1=-4$

$$= 9 + 16 - 12 + 32 + 3$$

$$= 48$$

$\therefore$ (3) is the correct answer

71. The equation of the diameter of the circle $x^2 + y^2 + 6x - 4y - 2 = 0$, passing through the point (2, –3) is

1. $x + 3y + 7 = 0$

2. $2x + y - 1 = 0$

3. $6x + 3y - 3 = 0$

4. $x + y + 1 = 0$

Soln. Given $x^2 + y^2 + 6x - 4y - 2 = 0$

$\therefore\ c \equiv (-3, 2)$ and given $p \equiv (2, -3)$

$\therefore$ equation of CP is given by

$$\frac{y-2}{x+3} = \frac{-5}{5} \Rightarrow \frac{y-2}{x+3} = -1$$

$$-(x+3) = y - 2 \Rightarrow x + y + 1 = 0$$

$\therefore$(4) is the answer

72. If one common tangent can be drawn to the circles $x^2 + y^2 - 2x - 4y - 20 = 0$ and $(x+3)^2 + (y+1)^2 = p^2$, then $p =$

1. 20
2. 16
3. 49
4. 10

Soln

Given

$x^2+y^2-2x-4y-20=0$ & $(x+3)^2+(y+1)^2=p^2$

$c_1 \equiv (1, 2)$ $\qquad$ $c_2 \equiv (-3, -1)$

$r_1 = \sqrt{1+4+20}$ $\qquad$ $r_2 = p$

$c_1c_2 = \sqrt{(-3-1)^2+(-1-2)^2}$ $\quad \therefore c_1c_2 = p-5$

$= 5$ $\qquad$ $5 = p-5$

$p = 10$

$\therefore$ (4) is the correct answer

73. If $x=7$ touches the circle $x^2+y^2-4x-67-12=0$, then the co-ordinates of the point of contact is

1. (7, 3)
2. (7, 4)
3. (7, 8)
4. (7, 2)

Soln.

Putting $x = y$ **in the given circle equation**

$$49 + y^2 - 28 - 6y - 12 = 0$$

$$y^2 - 6y + 9 = 0 \Rightarrow (y-3)^2 = 0 \Rightarrow y = 3$$

$\therefore$ **point of contact is (7, 3)**

$\therefore$ (1) is the correct answer

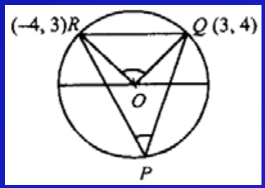
74. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3, 4) and (–4, 3) respectively then $Q\hat{P}R$ is

1. $\pi/2$
2. $\pi/3$
3. $\pi/4$
4. $\pi/6$

Slope of $OR = \frac{3}{-4} = m_1$

Slope of $OQ = \frac{4}{3} = m_2$

Clearly $R\hat{O}Q = 90^0$ $(m_1 m_2 = -1)$



$\therefore R\hat{P}Q = \frac{1}{2} R\hat{O}Q \Rightarrow R\hat{P}Q = \frac{\pi}{4}$

$\therefore$ (3) is the correct answer

75. If two lines $3x-2y-8=0$ and $2x-y-5=0$ lie along two diameters of a circle which touches the x – axis then the equation of the circle is

1. $(x-2)^2+(y-1)^2=1$

2. $(x+2)^2+(y-1)^2=1$

3. $(x-2)^2+(y+1)^2=1$

Soln. The point of intersection of the

lines $3x - 2y - 8 = 0$

$2x - y - 5 = 0$

is the centre i.e., $C = (2, -1)$

Circle touches x – axis $\Rightarrow r = |-1| = 1.$

$\therefore$ Equation is $(x-2)^2 + (y+1)^2 = 1$