COMPLEX NUMBERS

SYNOPSIS

1. A number of the form .

$z = x + iy$ is said to be complex number where $x, y \in R$ and $i = \sqrt{-1}$ imaginary number.

2. $i^{4n} = 1$, n is an integer.

3. In $z = x + iy$, x is called real part and y is called imaginary part .

4. If $z = x + iy$ then modulus of z is $|z| = \sqrt{x^2 + y^2}$

5. If z_1 and z_2 are any two complex numbers, then

$$|z_1 z_2| = |z_1|\,|z_2| \quad \text{and} \quad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

7)

π – α	0 α
-(π – α)	- α

Amplitude at the origin is not defined.

8) $\arg(z_1 z_2 \ldots z_n) = \arg z_1 + \arg z_2 + \ldots\ldots + \arg z_n$

$\arg \dfrac{z_1}{Z_2} = \arg z_1 - \arg z_2$

9) $e^{i\theta} = \cos\theta + i\sin\theta$

10) $(1+i)^2 = 2i$

$(1-i)^2 = -2i$

$\omega^{3n} = 1$

11) $(\sin\theta + i\cos\theta)^n = \sin n\theta + i\cos n\theta$ only when $n = 1 \pmod 4$

1) INTERGAL POWER OF IOTA, EQUALITY OF COMPLEX NUMBERS

Q.1) If $\left(\frac{1-i}{1+i}\right)^{200} = a + ib$

a) a = 2   b = -1

b) a = 1   b = 0

c) a = 0   b = 1

d) a = -1   b = 2

Sol: $(-i)^{200} = a + ib$

$(i^4)^{50} = a + ib$

$1 = a + ib$

$1 + 0i = a + ib$

$a=1$ & $b=0$

2) The sum of the series $i^2 + i^4 + i^6 +$ --------(2n + 1) terms is

a) 0 b) n c) 1 d) -1

sol: -1 +1-1+...........(2n+1)terms (odd number of terms) = -1

Q. 3) If $(x + iy)^{1/3} = 2 + 3i$, then $3x + 2y$ is equal to

a) -20 b) -60 c) -120 d) 60

Sol : $x + iy = (2+3i)^3$

$= -46+9i$

$3x+2y = -138+18$

$= -120$

Q.4) The value of $\sum_{n=0}^{\infty} \left[\frac{2i}{3}\right]^{n}$ is

a) $\frac{9+6i}{13}$ b) $\frac{9-6i}{3}$ c) $9+6i$ d) $9-6i$

Sol $= \frac{1}{1-\frac{2i}{3}}$

$= \frac{9+6i}{13}$

Q.5) The complex number $\frac{2^n}{(1+i)^{2n}} + \frac{(1+i)^{2n}}{2^n}$

, $n \in I$ is equal to

a) 0 b) 2 c) $\{1 + (-1)^n\} i^n$ d) $\{1 - (-1)^n\} i^n$

Sol: $\frac{2^n}{2^n i^n} + \frac{2^n i^n}{2^n}$

$= \frac{i^n}{i^{2n}} + i^n$

$= i^n[(-1)^n + 1]$

(II) Conjugate, Modules and Argument of complex numbers.

6) The argument of the complex number $\dfrac{13-5i}{4-9i}$ is

a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{5}$ d) $\dfrac{\pi}{6}$

Sol: $\dfrac{13-5i}{4-9i} \times \dfrac{4+9i}{4+9i}$

$= \dfrac{97+97i}{97}$

$z = 1 + i$

$\text{Arg}z = \tan^{-1}\underline{1} = \pi/4$

7) If $z = \sqrt{3} + i$, then the argument of $z^2 e^{z-i}$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $e^{\pi/6}$ d) $\frac{\pi}{3}$

Sol: $2\arg z + \arg e^{\sqrt{3}}$

$2.\pi/6 + 0$

$= \pi/3$

8) The modulus of the complex number z,such that

$|Z+3-i|=1$ and arg z = π is equal to

a) 1 b) 3 c) 9 d) 4

Sol: $\sqrt{(x+3)^2 + (y-1)^2} = 1$

and argz = π

y =0

X = - 3 & $|z|$ =3

9)The solution of the equation $|z|$ - z = 1 +2i is

a) $\frac{3}{2} + 2i$ b) $\frac{3}{2} - 2i$ c) 3 – 2i d) 3 + 2i

Sol: $\sqrt{x^2 + y^2} - x - iy = 1 + 2i$

$\sqrt{x^2 + y^2} - x = 1$ & $y = -2$

$\sqrt{x^2 + y^2} = 1 + x$

$x = \frac{3}{2}$

$z = \frac{3}{2} - 2i$

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10) If $\frac{5z_2}{11z_1}$ is purely imaginary, then the value of

$$\left|\frac{2Z_1+3Z_2}{2Z_1-3Z_2}\right|$$ is

a) $\frac{37}{33}$ b) 2 c) 1 d) 3

Sol: $\dfrac{5z_2}{11z_1} = iy$

$\dfrac{z_2}{z_1} = \dfrac{11}{5}iy$

$= \left|\dfrac{2 + 3(11/5)iy}{2 - 3(11/5)iy}\right|$

$= 1$

Hence c is the correct answer

11) If z is any complex number and $\bar{z}$ is its conjugate , then $\dfrac{\bar{z}}{|z|^2}$ =

a) $\dfrac{1}{z}$ b) $\dfrac{-1}{z}$ c) $\dfrac{1}{z}$ 4) $\dfrac{-1}{z}$

Sol: $\dfrac{\bar{z}}{|z|^2} = \dfrac{\bar{z}}{z\bar{z}} = \dfrac{1}{z}$

12) The complex number sinx +icos2x and Cosx -isin2x are conjugate to each other for

a) $x = n\pi$ b) $x = (n+1/2)\pi$

c) $x = 0$ d) No value of x.

Sol: $\sin x + i\cos 2x = \overline{\cos x - i\sin 2x}$

$\tan x = 1$ & $\tan 2x = 1$

Which is not possible for any values of x

13) If $z = r\,[\cos\theta + i\sin\theta]$, then the value of

$$\frac{z}{\bar{z}} + \frac{\bar{z}}{z} \text{ is}$$

a) $\cos 2\theta$ b) $2\cos 2\theta$ c) $2\cos\theta$ d) $2\sin\theta$

Sol: $\dfrac{z}{\bar{z}} + \dfrac{\bar{z}}{z} = \dfrac{re^{i\theta}}{re^{-i\theta}} + \dfrac{re^{-i\theta}}{re^{i\theta}}$

$= e^{i(2\theta)} + e^{-i(2\theta)}$

$= \cos 2\theta + i\sin 2\theta$

$+\cos 2\theta - i\sin 2\theta$

$= 2\cos 2\theta$

14) If $z = re^{i\theta}$, then $\left|e^{iz}\right| = 0$

a) $e^{r}\sin\theta$ b) $e^{-r}\sin\theta$ c) $e^{-r}\cos\theta$ d) $e^{r}\cos\theta$

Sol: $z=r[\cos\theta \quad +i\sin\theta]$

$iz=r[-\sin\theta+i\cos\theta]$

$e^{iz}=e^{-r\sin\theta}.e^{r\cos\theta i}$

$z = e^{-r\sin\theta}$

15) If $(\sqrt{5} + \sqrt{3}i)^{33} = 2^{49}z$, then modulus of the complex number z is equal to

a) 1 b) $\sqrt{2}$ c) $2\sqrt{2}$ d) 4

Sol: $2^{99/2} = 2^{49}|z|$

$|z| = \sqrt{2}$

Real and imaginary parts of complex numbers

16) If $z = \overline{z}$ then

a) z is purely real
b) z is purely imaginary
c) Real part of z = imaginary part of z
d) z is a complex number.

Sol: $z = \overline{z}$

$y = 0$

$z = x$ is purely real .

17) The imaginary part of $e^{e^{i\theta}}$ is

a) θ

b) $e^{\cos\theta}\sin(\sin\theta)$

c) $e^{\cos\theta}\cos(\sin\theta)$

d) $e^{e^{i\theta}}$

Sol: $e^{\cos\theta + i\sin\theta}$

$= e^{\cos\theta}[\cos(\sin\theta)+i\sin(\sin\theta)]$

18) Let $z = \dfrac{11 - 3i}{1 + i}$, If α is real numbers, such that z-iα is real, then the value of α is

a) 4 b) -4 c) 7 d) -7

Sol: $z = 4 - 7i$

Since $z - i\alpha$ is real

$4 - 7i - i\alpha$ is real

If $\alpha = -7$

19)If z is a complex number such that Re(z) = Im(z), then

a) $Rez^2 = 0$

b) $Imz^2 = 0$

c) $Rez^2 = Imz^2$

d) $Rez^2 = - Imz^2$

Sol: $z = x+iy$

$z^2=(x^2-y^2) + 2xyi$

If $x = y$

$Rez^2 =0$

20) Real part of log $(1 - i\sqrt{3})$

a) 1 b) log 2 c) $-\pi/3$ d) $\pi/2$

Sol: $\log 2e^{-\pi/3i}$

$= \log 2 - \pi/3i$

DE MOIVRE'S THEOREM AND ROOTS OF UNITY

21) If $z_r = \cos\frac{r\alpha}{n^2} + i\sin\frac{r\alpha}{n^2}$, where

$r = 1, 2, \ldots\ldots\ldots\ldots n$

$\lim_{n \to \infty} z_1 z_2 z_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots z_n =$

a) $\cos\alpha + i\sin\alpha$

b) $\cos\frac{\alpha}{2} - i\sin\frac{\alpha}{2}$

c) $e^{i\alpha/2}$

d) $3\sqrt{e^{i\alpha}}$

Sol: $\lim_{n\to\infty} \operatorname{cis}\alpha\left(\frac{n^2+n}{2n^2}\right)$

$= \operatorname{cis}\alpha\left(\frac{1}{2}\right)$

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22) $(\sin\theta + i\cos\theta)^{17} =$

a) $\sin 17\theta - i\cos 17\theta$

b) $\cos 17\theta + i\sin 17\theta$

c) $\sin 17\theta + i\cos 17\theta$

d) $\cos 17\theta - i\sin 17\theta$

Sol: 4 | 17 -1

23) The product of 10^{th} roots of 7 is

a) $7 \text{ cis} \frac{\pi}{10}$ b) 7 c) -7 d) 14

Sol: $(-1)^{n+1}z$

$=-7$

24) If α is the cube roots of -1 then

a) $1 + \alpha + \alpha^2 = 0$

b) $\alpha^2 - \alpha + 1 = 0$

c) $\alpha^2 + 2\alpha + 1$

d) $\alpha^2 + 2\alpha - 1 = 0$

Sol: $\alpha^3 + 1^3 = 0$

$(\alpha + 1)(\alpha^2 - \alpha + 1) = 0$

25) $$\left(\frac{1+\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}}\right)^{80} =$$

a) -1 b) 0 c) 1 d) 2

Sol: $z = \left(\text{cis}\frac{\pi}{8}\right)^{80}$

26) If ω is a complex cube roots of unity, then the value of

$$\omega + \omega^{1/2+3/8+9/32+27/128+\ldots\ldots\ldots}$$

a) 1 b) ω c) ω2 d) -1

Sol: $\dfrac{1/2}{1-\frac{3}{4}} = 2$

G.E $= \omega + \omega^2$

$= -1$

27) if $iz^4 + 1 = 0$ then z can take the value

a) $\frac{1 + I}{\sqrt{2}}$ b) $cis\frac{\pi}{8}$ c) $\frac{1}{4i}$ d) i

Sol: $z^4 = i$

$Z = (cis\pi/2)^{1/4}$

28) If $x = \alpha + \beta$, $y = \alpha\omega + \beta\omega^2$, $z = \alpha\omega^2 + \beta\omega$ Where ω is cube roots of unity, then value of xyz is

a) $\alpha^2 + \beta^2$ b) $\alpha^2 - \beta^2$
c) $\alpha^3 + \beta^3$ d) $\alpha^3 - \beta^3$

Sol: $xyz = (\alpha + \beta)(\alpha\omega + \beta\omega^2)(\alpha\omega^2 + \beta\omega)$
$= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

29) The complex numbers 1 , -1, $i\sqrt{3}$ form a triangle which is

a) right angled
b) isosceles
c) equilaterial
d) isosceles right angled

Sol: The given complex numbers are represented by the points A(1,0),B(-1,0) & C(0,$\sqrt{3}$)

AB=BC=AC=2

30) If $\left|\dfrac{z-2}{z-3}\right| = 2$ represents a circle, then its radius is equal to

a) $\dfrac{1}{3}$ b) $\dfrac{3}{4}$ c) $\dfrac{2}{3}$ d) 1

S0l: $z = x+iy$

$|z-2|^2 = 4|z-3|^2$

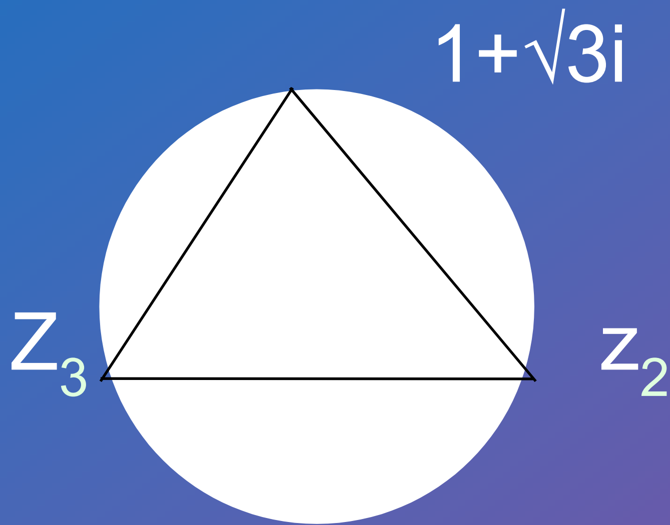
$x^2 + y^2 - \dfrac{20}{3}x + \dfrac{32}{3} = 0$

Radius $= \dfrac{2}{3}$

31) Suppose z_1, z_2 , z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$ if $z_1 = 1 + i\sqrt{3}$ and z_1 , z_2, z_3 are in Clock wise sense, then z_2 is

a) $1 - \sqrt{3}i$ b) 2 c) $-1 + \sqrt{3}i$ d) $-1 - \sqrt{3}i$

Sol:



$\text{amp}z_1 = \text{amp}(1+\sqrt{3}i) \quad = \pi/3$

$\text{amp}z_2 = \pi/3 - 2\pi/3 = -\pi/3 \; \& \; |z| = 2$

$z_2 = 2\text{cis}(-\pi/3) = 1 - i\sqrt{3}$

OTHER AND MIXED TYPES :-

32) $(1 +i)^6 + (1-i)^3 =$

a) $2 + i$ b) $2 -10i$ c) $-2 + I$ d) $–2 -10i$

Sol: $[(1 +i)^2]^3 + [(1 -i)^2](1-i)$

$= (2i)^3 + (-2i)(1-i)$

$= -2 -10i$

33) If $x+iy = \dfrac{1}{1+\cos\theta+i\sin\theta}$ then x^2=

a) $\dfrac{1}{2}$ b) $\dfrac{1}{3}$ c) $\dfrac{1}{4}$ d) $\dfrac{1}{8}$

Sol: $x+iy = \dfrac{1}{2\cos^2\theta/2 + 2i\sin\theta/2\cos\theta/2}$

$$= \frac{1}{2\cos\theta/2\,[\cos\theta/2 + i\sin\theta/2\,]}$$

$$x = \frac{1}{2}\sec\theta/2\,\cos\theta/2 \;=\; \frac{1}{2}$$

$x^2 = 1/4$

Henc c is the correct answer.

34) If n is any integar , i^n is

a) 1, -1, i, -I b) i, -I c) 1,-1 d) i

Sol: n = 1,2 ,3,4, so on.

Then we get , 1, -1, i, -I

35)If $n = 4m + 3$, m is an integer the i^n is

a) i b) -i c) -1 d) 1

Sol: $(i^4)^m i^3$

$= -i$

36) If $z = \frac{\sqrt{3} + i}{2}$, then z^{69} is equal to

a) $-i$ b) i c) 1 d) -1

37) The value of $\left[\dfrac{1 + i\sqrt{3}}{1 - i\sqrt{3}}\right]^6 + \left[\dfrac{1 - i\sqrt{3}}{1 + i\sqrt{3}}\right]^6$ is

a) 2 b) -2 c) 1 d) 0

38) If ω is a complex cube roots of unity then $\sin [(\omega^{10} + \omega^{23}) \pi - \pi/4]$ is equal to

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1i}{\sqrt{2}}$ c) 1 d) $\frac{\sqrt{3}}{2}$

39) The square root of $-7 + 24i$ is $x + iy$ then x

a) ± 1 b) ± 2 c) ± 3 d) ± 4

40) The value of $e^{2+i\pi/3} + e^{2-i\pi/3}$

a) 1 b) e^2 c) $2e^2$ d) $i\sqrt{3}$

42) If $\dfrac{1 + \text{icosə}}{2 + \text{icosə}}$ is purely real then ə =

a) nπ ± π/2

b) 2nπ ± π/2

c) nπ ± 1

d) 2nπ ± π

43) If $a = \text{cis}\alpha$, $b = \text{cis}\beta$, $c = \text{cis}\,\gamma$ and $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 1$

Then $\sum \sin(\alpha - \beta) =$

a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 1 d) 0

44) If $|z^2 - 1| = |z|^2 + 1$, then z lies on

a) a circle

b) $x = 0$

c) a parabola

d) $y = 0$

45) The value of $\dfrac{i^{592} + i^{590} + i^{588}}{i^{582} + i^{580} + i^{578}} + 1$ is

a) -1 b) 0 c) 1 d) 2i

46) The value of $\sum_{K=1}^{\infty}\left(\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right)$ is

a) 1 b) -1 c) i d) -i

47) The common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$ are

a) $-1, \omega$ b) $-1, \omega^2$ c) ω, ω^2 d) $1, \omega^2$

48) The equation $|(z-1)|^2 + |(z+1)|^2 = 4$ represents on the argand plane

a) a straight line
b) an ellipse
c) a circle with centre origin and radius 2
d) a circle with centre origin and radius unity

49) arg bi (b>0) is

a) π b) $\pi/2$ c) $-\pi/2$ d) 0

50) For the +ve integer n, the expression

$$(1 - i)^n \left(1 - \frac{1}{i}\right)^n =$$

a) 0 b) $2i^n$ c) 2^n d) 4^n

51) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} =$

a) $3\sqrt{3}i$ b) $-3\sqrt{3}i$ c) $i\sqrt{3}$ d) 3

52) Let $a_n = i^{(n+1)^2}$ where $i = \sqrt{-1}$ and $n = 1, 2, 3$.................... then the values of

$a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots\ldots\ldots + a_{25}$ is

a) 13 b) 13 + i c) 13 -i d) 12

53) The additive inverse of 1 -i is

a) 0 + 0i b) -1 + i c) -1 – i d) 1 - i

54) one of the value of $\left[\dfrac{1+i}{\sqrt{2}}\right]^{2/3}$

a) $\sqrt{3} + i$ b) $-i$ c) i d) $-\sqrt{3} + i$

55 $\dfrac{(\cos 2\theta + i\sin 2\theta)(\cos 75 + i\sin 75)(\cos 10 + i\sin 10)}{(\sin 15 - i\cos 15)}$

a) 0 b) -1 c) i d) 1

56) $\sqrt{i} - \sqrt{-i}$ is equal to

a) $\pm i\sqrt{2}$ b) $\frac{1}{\pm i\sqrt{2}}$ c) 0 d) $\pm\sqrt{2}$

57) The value of $\left[i^{99} + \left[\frac{1}{i} \right]^{25} \right]^{2}$ is

a) -4 b) 4 c) 2 d) -2

58) If $iz^3 + z^2 - z + i = 0$ then $|z| =$

a) 1 b) i c) -1 d) $-i$

59) If $\begin{vmatrix} 60 & -i & 1 \\ 7 & i & -1 \\ 7i & 1 & i \end{vmatrix} = x + iy$ then

a) $x=3, y=1$ b) $x=1, y=3$

c) $x=0, y=3$ d) $x=0, y=0$

60) If $z_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$, r= 1, 2,3 ......then

$z_1, z_2, z_3 \ldots\ldots \infty$

a) i

b) -i

c) 1

d) -1

61) The value of $\log i^i$ is

a) ω b) $-\omega^2$ c) $\pi/2$ d) $-\pi/2$

62) $\left[\frac{-1 + i\sqrt{3}}{2}\right]^{65n} + \left[\frac{-1 - i\sqrt{3}}{2}\right]^{65n} =$

a) 1 b) 2 c) 3 d) ω^2

63) Let z, ω be complex numbers , suchthat

$\overline{z} + i\,\overline{\omega} = 0$ and $\arg z\omega = \pi$

Then arg z =

a) $5\pi/4$ b) $\pi/2$ c) $3\pi/4$ d) $\pi/4$

64) If ω is a complex cube root of unity, then $(3 + \omega + 2\omega^2)^4$ equals

a) 1 b) ω^2 c) 9ω d) $9\omega^2$

65 The value of amp $(i\,\omega)$ + amp $(i\,\omega^2)$ where $i = \sqrt{-1}$ and ω = non zero real number

a) 0 b) $\pi/2$

c) π d) $-\pi$

66) If $x + \frac{1}{x} = 2\cos\theta$, & $y + \frac{1}{y} = 2\cos\Phi$ Then $2\cos(\theta + \Phi) =$

a) $\frac{1}{x^2+xy}$

b) $xy + \frac{1}{xy}$

c) $(x+\frac{1}{x})(y+\frac{1}{y})$

d) $xy - \frac{1}{xy}$

67. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7 =$

a) 128ω b) -128ω c) $128\omega^2$
d) $-128\omega^2$

68). If α is the complex number such that $\alpha^2 + \alpha + 1 = 0$ then α^{31} is

a) 1 b) 0 c) α^2 d) α

69).The curve represents $Re(z^2) = 4$ is

a parabola

b) an ellipse

c) circle

d) a rectangular hyperbola.

69).The conjugate of (1+2i)(2-3i) is

a) - 4 + i

b) -4 – i

c) 8 +i

d) 8 - i

70).If $\left[\frac{1+i}{1-i}\right]^3 - \left[\frac{1-i}{1+i}\right]^3 = x+iy$

Then (x, y) =

a) (0,2) b) (-2,0) c) (0 ,2) d) (0,0)

71). If $x = -1 - i\sqrt{3}$ then x^3 is

a) 8 b) -8 c) 1 d) -1

72). The value of $\sqrt{2i}$

a) $1+i$ b) $-1-i$ c) $-\sqrt{2}i$ d) $\pm(1+i)$

73)Let $z=x+iy$ and $z.\bar{z}=0$

Then

a) Re(z)=0 b) z=0

c) Ip(z)=0 d) z=-1

74) If $\omega \neq 1$, the least +ve value of n, for Which $(1 + \omega^2)^n = (1 + \omega^4)^n$

a) 1 b) 2 c) 3 d) 4

75). The amp of $(-i)^5$

a) $-\pi$ b) π c) $\pi/2$ d) $-\pi/2$

KEY ANSWERS

Q.1)	(B)	Q.28)	(C)	Q.55)	(B)
Q.2)	(D)	Q.29)	(C)	Q.56)	(A)
Q.3)	(C)	Q.30)	(C)	Q.57)	(A)
Q.4)	(A)	Q.31)	(A)	Q.58)	(A)
Q.5)	(C)	Q.32)	(D)	Q.59)	(D)
Q.6)`	(B)	Q.33)	(C)	Q.60)	(A)
Q.7)	(D)	Q.34)	(A)	Q.61)	(D)
Q.8)	(B)	Q.35)	(B)	Q.62)	(B)
Q.9)	(B)	Q.36)	(A)	Q.63)	(C)
Q.10)	(C)	Q.37)	(A)	Q.64)	(D)
Q.11)	(A)	Q.38)	(B)	Q.65)	(C)
Q.12)	(D)	Q.39)	(A)	Q.66)	(D)
Q.13)	(B)	Q.40)	(C)	Q.67)	(D)
Q.14)	(B)	Q.41)	(B)	Q.68)	(D)
Q.15)	(B)	Q.42)	(B)	Q.69)	(D)
Q.16)	(A)	Q.43)	(D)	Q.70)	(C)
Q.17)	(B)	Q.44)	(B)	Q.71)	(A)
Q.18)	(D)	Q.45)	(B)	Q.72)	(B)
Q.19)	(A)	Q.46)	(C)	Q.73)	(B)
Q.20)	(B)	Q.47)	(C)	Q.74)	(C)
Q.21)	(C)	Q.48)	(D)	Q.75)	(D)
Q.22)	(C)	Q.49)	(B)		
Q.23)	(C)	Q.50)	(C)		
Q.24)	(B)	Q.51)	(A)		
Q.25)	(C)	Q.52)	(A)		
Q.26)	(D)	Q.53)	(B)		
Q.27)	(B)	Q.54)	(C)		