SOLUTIONS

1) Ans. (2)

$\vec{a}+\vec{b}+\vec{c} = (i+j-2k) + (-i+2j+k) + (i-2j+2k) = i+j+k$

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{1+1+1} = \sqrt{3}$

Unit vector $\hat{n} = \dfrac{\vec{a}+\vec{b}+\vec{c}}{|\vec{a}+\vec{b}+\vec{c}|} = \dfrac{i+j+k}{\sqrt{3}}$

2) Ans. (4)

Volume of the parallelopiped $= \vec{a}.(\vec{b}\times\vec{c})$

$$\vec{a}.(\vec{b}\times\vec{c}) = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 2 & -2 \\ 6 & 1 & -1 \end{vmatrix}$$

$= -22$

Therefore volume = |-22| = 22 cubic units.

3) Ans. (2)

$\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|\,|\vec{b}|}$

$\vec{a}.\vec{b} = (2,-3,6).(4,8,-8) = 8-24-48 = -64$

$|\vec{a}| = \sqrt{4+9+36} = \sqrt{49} = 7$

$|\vec{b}| = \sqrt{16+64+64} = \sqrt{144} = 12$

$\cos\theta = -64/(7)(12) = -16/21$

4) Ans. (2)

$$[\, i-j,\ j-k,\ k-i] = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} [i, j, k]$$

$= 0(1) = 0$

5) Ans. (3)

$$[\vec{a},\vec{b},\vec{c}] = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & -1 \end{vmatrix}$$

$= 1(-1) -2(-4)+0(\ )$

$= 7$

6) Ans.(3)

The unit vector is $\cos\alpha\cos\beta\ i + \cos\alpha\sin\beta\ j + \sin\alpha\ k$

Since magnitude $= \sqrt{\cos^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha(\cos^2\beta+\sin^2\beta)+\sin^2\alpha}$

$= \sqrt{\cos^2\alpha+\sin^2\alpha}$

$= 1$

7) Ans. (2)

$\vec{a} = 6i +2j + k$ $\vec{b} = i-j+2k$ and $\vec{c} = 5i +3j-k$

clearly $\vec{b} +\vec{c}-\vec{a}$ is a null vector.

8) Ans. (1)

In the vector addition of vectors of the triangle ABC whose sides are AB, BC, CA is $\vec{AB} +\vec{BC} + \vec{CA}$, here the initial and the terminal points coincide.

$\rightarrow$

Therefore the sum of the vectors is 0.

9) Ans. (2)

$\vec{a} \cdot \vec{b} = (3, a, -1) \cdot (1, 2, 1) = 6$

$= 3+2a-1 = 6$

$a = 2$

10) Ans. (2)

$\vec{a} \cdot (\vec{b} + \vec{c}) = (2,-1,3) \cdot [(1,2,1) + (2,1,1)]$

$= (2, -1, 3) \cdot (3,3,2)$

$= 6-3+6$

$= 9$

11) Ans. (4)

Let A= (1,3,-5) , B= (4,7,7) Therfore $\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AB} = (3, 4, 12)$

$|\vec{AB}| = \sqrt{9 + 16 + 144} = 13$

Direction cosines are $\cos\alpha = 3/13$, $\cos\beta = 4/13$,

$\cos\gamma = 12/13$

12) Ans. (1)

w.k.t $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$(1-\cos^2\alpha) + (1-\cos^2\beta) + (1-\cos^2\gamma)$

$3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$3 - 1 = 2$

13) Ans. (3)

$|\vec{a}| = 2$, $|\vec{b}| = 1$, $\vec{a} \cdot \vec{b} = 1$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$1 = (2)(1)\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \pi/3$$

14) Ans. (1)

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$= 1+1+2\cos\theta$$

$$= 2 + 2(-1/2)\text{ , if } \theta = 120^0$$

$$= 2 - 1$$

$$= 1$$

15) Ans. (2)

$$(2\vec{a} + \vec{b}) \times (\vec{a} + 2\vec{b}) = 2(\vec{a}\times\vec{a}) + 4(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{a}) + 2(\vec{b}\times\vec{b})$$

$$= 0 + 4(\vec{a}\times\vec{b}) - (\vec{a}\times\vec{b}) + 0$$

$$= 3(\vec{a}\times\vec{b})$$

16) Ans. (2)

$$[\, i+j,\ j+k,\ k+i\,] = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} [i, j, k]$$

$$= 1(1) - (-1) + 0(\ )$$

$$= 2$$

17) Ans. (3)

A vector of magnitude 15 units in the direction of

$$2\vec{a} + 3\vec{b} - \vec{c} \text{ is } 15\,\hat{n} = 15(2\vec{a} + 3\vec{b} - \vec{c})/|2\vec{a} + 3\vec{b} - \vec{c}|$$

$$2\vec{a} + 3\vec{b} - \vec{c} = 2(1,-2) + 3(2,1) - (3,-1)$$

$$= (5, 0)$$

$|2a + 3b - c| = \sqrt{25+0} = 5$

Therefore required vector = 15(5,0)/5 = 15(1,0).

18) Ans. (3)

$C = (\text{projection of } \vec{b} \text{ on } \vec{a}) / (\text{projection of } \vec{a} \text{ on } \vec{b})$

$C = [(\vec{a}.\vec{b})/|\vec{a}|]/[(\vec{a}.\vec{b})/|\vec{b}|]$

$$C = |\vec{b}|/|\vec{a}| = \frac{\sqrt{4+4+1}}{\sqrt{4+9+36}} = \frac{3}{7}$$

19) Ans. (2)

$\lambda(i+j+k)$ is a unit vector $\Rightarrow |\lambda(i+j+k)| = 1$

$\sqrt{\lambda^2(1+1+1)} = 1$

$\lambda\sqrt{3} = 1$

$\lambda = 1/\sqrt{3}$

20) Ans. (3)

$\vec{AB}$ perpendicular to BC, means $(\vec{AB}) . (\vec{BC}) = 0$

$\vec{AB} = \vec{OB} - \vec{OA} = (2i+k) - (i+xj+k) = i - xj$

$\vec{BC} = \vec{OC} - \vec{OB} = (-i+j+k) - (2i+k) = -3i+j$

$(\vec{AB}).(\vec{BC}) = (1,-x,0) . (-3,1,0) = 0$

$= -3 - x = 0$

$x = -3$

21) Ans. (3)

$$\text{Required vector} = 10\frac{[\vec{a} \times (\vec{b} \times \vec{c})]}{|[\vec{a} \times (\vec{b} \times \vec{c})]|}$$

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} . \vec{c})\vec{b} - (\vec{a} . \vec{b})\vec{c}$

$= (1+2+1)(2i-j-k) - (2-1+1)(i+2j-k)$

$= (8i-4j-4k) - (2i+4j-2k)$

$= 6i-8j-2k$

$|\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{36+64+4} = \sqrt{104} = 2\sqrt{26}$

Required vector = $10(6i-8j-2k)/2\sqrt{26} = 10(3i-4j-k)/\sqrt{26}$

22) Ans. (2)

the value of $\dfrac{\vec{a} \cdot (\vec{b} \times \vec{c})}{\vec{c} \cdot (\vec{a} \times \vec{b})} + \dfrac{\vec{b} \cdot (\vec{a} \times \vec{c})}{\vec{c} \cdot (\vec{a} \times \vec{b})}$

= 1 + (-1) (The value of the scalar triple product will not alter if the vectors are interchanged in a cyclic order.

The sign of the scalar triple product will be changed if any two vectors are interchanged)

= 0

23) Ans. (1)

Given : $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$, **also** $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

(since the vectors are mutually perpendicular)

w.k.t $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$= 1 + 1 + 1 + 2(0+0+0)$

$= 3$

Therefore $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$

24) Ans. (1)

$\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

since $\vec{a} \neq 0$, $\vec{b} - \vec{c} = 0$ **i.e.,** $\vec{b} = \vec{c}$

25) Ans. (2)

Clearly $\vec{a}, \vec{b}, \vec{c}$ **are the sides of the triangle**

ABC, we have $\vec{a} + \vec{b} + \vec{c} = 0$

$$\vec{a} + \vec{b} = -\vec{c}$$

$$\vec{a} \times (\vec{a} + \vec{b}) = -(\vec{a} \times \vec{c})$$

$$\vec{a} \times \vec{a} + \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$$

$$\vec{a} \times \vec{b} = \vec{c} \times \vec{a}$$

similarly, $\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$, $\therefore$ $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

26) Ans. (4)

since $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$|\vec{a} \times \vec{b}| = \sin\theta$

27) Ans. (4)

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta > 0$ if $0 \leq \theta < \pi/2$.

(since $\cos\theta > 0$ for $0 \leq \theta < \pi/2$

28) Ans. (2)

Let $\vec{b} = (k, 2k, -k)$ as b is collinear with a

also $\vec{a} \cdot \vec{b} = 5 \Rightarrow k + 4k + k = 5 \Rightarrow k = 5/6$

$\therefore \vec{b} = (1/6)(5,10,-5)$

29) Ans. (4)

$|\vec{a} \cdot \vec{b}| = |ab\cos\theta| \leq ab$. (since $|\cos\theta| \leq 1$)

30) Ans.(1)

Both the vectors $\vec{a}$ and $\vec{b}$ cannot be parallel and perpendicular unless either $\vec{a} = 0$ or $\vec{b} = 0$

31) Ans.(1)

$\vec{AB} = \vec{OB} - \vec{OA} = -6i-2j+3k$, $\vec{BC} = -2i+3j-6k$

$\vec{CD} = 6i+2j-3k$, $\vec{DA} = 2i-3j+6k$

Clearly, $|\vec{AB}| = |\vec{BC}| = |\vec{CD}| = |\vec{DA}| = 7$

$\therefore$ABCD is either a square or a rhombus.

But, $\vec{AB}.\vec{BC} = 12-6-18 = -12 \neq 0$.

Therefore $\vec{AB}$ is not perpendicular to $\vec{BC}$.

Hence ABCD is a rhombus

32) Ans. (1)

Since $\vec{p}$ is perpendicular to each of $\vec{a}, \vec{b}, \vec{c}$.

Therefore $\vec{a}, \vec{b}, \vec{c}$ are coplanar. $\therefore [\vec{a}, \vec{b}, \vec{c}] = 0$

33) Ans. (4)

$|\vec{a} + \vec{b}| < 1 \Rightarrow |\vec{a} + \vec{b}|^2 < 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta < 1$

$\Rightarrow 1 + 1 + 2\cos\theta < 1$

$\Rightarrow \cos\theta < -1/2$

$\Rightarrow 2\pi/3 < \theta < \pi$

34) Ans. (1)

$\vec{a}.[(\vec{b}+\vec{c}) \times (\vec{a}+\vec{b}+\vec{c})] = \vec{a}.[(\vec{b}+\vec{c}) \times \vec{a}] = 0$

(since cross product of two identical vectors is 0.
The value of scalar triple product is 0 if any two vectors are identical.)

35) Ans. (1)

Since $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$$

$4\,\vec{a}.\,\vec{b}=0 \Rightarrow \vec{a}$ perpendicular to $\vec{b}$

36) Ans. (3)

$\vec{a} \times \vec{c} = \vec{c} \times \vec{b} \Rightarrow (\vec{a}-\vec{c}) \times \vec{b} = 0 \Rightarrow \vec{a}-\vec{c} \parallel \vec{b}, \Rightarrow \vec{a} - \vec{c} = \lambda\vec{b}$

37) Ans. (1)

The scalar product and also the vector product of a scalar and vector is not possible.

38) Ans.(4)

$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}.\,\vec{c})\,\vec{b} - (\vec{b}.\,\vec{c})\vec{a}$$

$$[(\vec{a}\times\vec{b})\times\vec{c}].\vec{c} = (\vec{a}.\vec{c})(\vec{b}.\vec{c}) - (\vec{b}.\vec{c})(\vec{a}.\vec{c}) = 0$$

$\Rightarrow$ $(\vec{a}\times\vec{b})\times\vec{c}$ is perpendicular to $\vec{c}$.

39) Ans. (4)

$$\vec{a} \times \vec{b} = \vec{c},\ \vec{b} \times \vec{c} = \vec{a}$$

$\Rightarrow (\vec{b}\times\vec{c})\times\vec{b} = \vec{a} \times \vec{b} = \vec{c}$

$\Rightarrow (\vec{b}\,.\,\vec{b})\vec{c} - (\vec{c}\,.\,\vec{b})\vec{b} = \vec{c}$

$\Rightarrow (\vec{b}\,.\,\vec{b}) = 1,\ (\vec{c}\,.\,\vec{b}) = 0$

$\Rightarrow |\vec{b}|^2 = 1$ i.e., $|\vec{b}| = 1$ therefore $\vec{b}$ is a unit vector.

40) Ans. (2)

Since $[\vec{e}_1', \vec{e}_2', \vec{e}_3'] = 1/\{[\vec{e}_1, \vec{e}_2, \vec{e}_3]\}$.

Therefore $[\vec{e}_1', \vec{e}_2', \vec{e}_3'][\vec{e}_1, \vec{e}_2, \vec{e}_3] = 1$

41) Ans. (1)

$$(\vec{a} - \vec{b}) \cdot [(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})]$$

$$= (\vec{a} - \vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}]$$

$$= \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{a}\cdot(\vec{c}\times\vec{a}) + \vec{a}\cdot(\vec{a}\times\vec{b}) - \vec{b}\cdot(\vec{b}\times\vec{c}) - \vec{b}\cdot(\vec{c}\times\vec{a}) - \vec{b}\cdot(\vec{a}\times\vec{b})$$

$= \vec{a}\cdot(\vec{b}\times\vec{c}) - \vec{b}\cdot(\vec{c}\times\vec{a})$ (since the value of scalar triple product is 0 if any two vectors are identical.)

$= 0$ (The value of S.T.P will not alter if the vectors are interchanged in a cyclic order.)

42) Ans. (3)

Direction of zero vector can be any i.e., indeterminate.

43) Ans. (2)

Since $(\vec{a} + 3\vec{b})\cdot(2\vec{a} - \vec{b}) = -10$

$2\vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + 6\vec{b}\cdot\vec{a} - 3\vec{b}\cdot\vec{b} = -10$

$2\vec{a}^2 - 0 + 0 - 3\vec{b}^2 = -10$ (since $\vec{a} \perp \vec{b}$, $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a} = 0$)

$2|\vec{a}|^2 - 3|\vec{b}|^2 = -10$

$2(1) - 3|\vec{b}|^2 = -10 \Rightarrow |\vec{b}|^2 = 4 \Rightarrow |\vec{b}| = 2$

44) Ans.(2)

Required volume = $(1/6)\,[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}]$

Now $\vec{AB} = (2, -9, -1)$, $\vec{AC} = (-7, -2, 2)$ $\vec{AD} = (-2, -5, -1)$

$$\text{Reqd.volume} = (1/6)\begin{vmatrix} 2 & -9 & -1 \\ -7 & -2 & 2 \\ -2 & -5 & -1 \end{vmatrix}$$

$$= (1/6)\ [\ 2(12) + 9(11) - (31)]$$

$$= 46/3 \text{ cubic units.}$$

45) Ans. (1)

Let γ be the acute angle made by the line with the +ve direction of z – axis. Now $\cos\alpha = \cos 60^0 = ½$

$\cos\beta = \cos 120^0 = -1/2$.

Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore (1/4) + (1/4) + \cos^2\gamma = 1 \Rightarrow \cos^2\gamma = 1 - 1/2 = ½$

$\Rightarrow \cos\gamma = 1/\sqrt{2}$ **(since γ is acute)**

$\Rightarrow \gamma = 45^0$

46) Ans. (2)

Vectors are perpendicular if $2a + 3b - 4c = 0$

which is true if a=4, b=4, c=5.

47)

Let $\vec{c} = \vec{a} + \vec{b}$ where $\vec{a}, \vec{b}, \vec{c}$ are all unit vectors.

$\therefore \vec{c}^2 = (\vec{a} + \vec{b})^2 \Rightarrow |\vec{c}|^2 = |\vec{a} + \vec{b}|^2$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\,.\vec{b}$

$\Rightarrow 1 = 1 + 1 + 2\vec{a}.\vec{b} \Rightarrow 2\vec{a}.\vec{b} = -1$

$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b} \Rightarrow |\vec{a} - \vec{b}|^2 = 1 + 1 - (-1) = 3$

$|\vec{a} - \vec{b}| = \sqrt{3}$. Therefore Ans. (2)

48) Ans.(4) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0 \Rightarrow \vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular.

$\therefore \vec{a}\cdot(\vec{b} \times \vec{c}) = |\vec{a}||\vec{b}\times\vec{c}| \cos\theta$

$= |\vec{a}||\vec{b}||\vec{c}|\sin(\pi/2)\cos 0^0$

$= |\vec{a}||\vec{b}||\vec{c}|$

49) Ans. (3)

Area of triangle $= (1/2)\, |\vec{AB} \times \vec{AC}|$

$= (1/2)\, |(\vec{b}-\vec{a}) \times (\vec{c}-\vec{a})|$

$= (1/2)\, |\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}|$

50) Ans. (3) $\vec{a} \times (\vec{b} \times \vec{a})$ is perpendicular to $\vec{a}$.

51) Ans. (2) $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

If $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ then $\vec{a}\cdot\vec{b} = 0$

i.e., $\vec{a}$ is perpendicular to $\vec{b}$

52) Ans. (2) Since $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{a} - \lambda\vec{b}$

$(\vec{a} + \lambda\vec{b})\cdot(\vec{a} - \lambda\vec{b}) = 0$

$$|\vec{a}|^2 - \lambda^2|\vec{b}|^2 = 0$$

$$\Rightarrow 9 - \lambda^2(16) = 0 \Rightarrow \lambda = 3/4.$$

53) Ans. (1)

Since $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{b} + \vec{c})/\sqrt{2}$

$\therefore (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (1/\sqrt{2})\, \vec{b} + (1/\sqrt{2})\, \vec{c}$

$\therefore \vec{a} \cdot \vec{c} = (1/\sqrt{2})$, $\vec{a} \cdot \vec{b} = (-1/\sqrt{2})$ (since $\vec{a},\vec{b},\vec{c}$ are non- coplanar)

$|\vec{a}||\vec{b}|\cos\theta = (-1/\sqrt{2})$

$\cos\theta = (-1/\sqrt{2})$ (since $\vec{a}$, $\vec{b}$ are unit vectors)

$\cos\theta = (-1/\sqrt{2}) = \cos(3\pi/4)$

$\therefore \theta = 3\pi/4$

54) Ans. (3)

$[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$

$= |\vec{u}| \cdot |\vec{v} \times \vec{w}| \cos\theta$ (where θ is the angle between $\vec{u}$ and $\vec{v} \times \vec{w}$)

$= 1 \cdot |\vec{v} \times \vec{w}|$ (since max. value of $\cos\theta$ is 1)

$= |(2i + j - k) \times (i + 3k)|$

$= |3i - 7j - k| \Rightarrow \sqrt{9+49+1} = \sqrt{59}$

55) Ans. (2)

$(\vec{a}+2\vec{b}) \cdot (5\vec{a}-4\vec{b}) = 0$

$\Rightarrow 5 - 4(\vec{a} \cdot \vec{b}) + 10(\vec{a} \cdot \vec{b}) - 8 = 0 \Rightarrow 6\vec{a} \cdot \vec{b} = 3 \Rightarrow \vec{a} \cdot \vec{b} = 1/2$

$\therefore$ angle between $\vec{a}$ and $\vec{b}$ is $\cos^{-1}(1/2) = 60^0$.

56) Ans. (2)

$|\vec{a}| = \sqrt{9 + 25} = \sqrt{34}$, $|\vec{b}| = \sqrt{36 + 9} = \sqrt{45}$

Now, $|\vec{c}| = |\vec{a} \times \vec{b}|$

$= |(3i-5j) \times (6i+3j)|$

$= |9k + 30k| = |39k| = 39$

$\therefore |\vec{a}| : |\vec{b}| : |\vec{c}| = \sqrt{34} : \sqrt{45} : 39$

57) Ans. (3)

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b}$

$= 1 + 1 + 2\cos\theta = 2(1 + \cos\theta)$

$= 2[2\cos^2(\theta/2)]$

$|\vec{a} + \vec{b}| = 2\cos(\theta/2)$

58) Ans. (3)

Volume of the paralellopiped formed by vectors

$$= V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 - a + a^3$$

$\frac{dV}{da} = -1 + 3a^2$, $\frac{d^2V}{da^2} = 6a$. For Max. or Min of V, $dV/da = 0$

$\Rightarrow a^2 = 1/3 \Rightarrow a = 1/\sqrt{3}$, $d^2V/da^2 = 6a > 0$ for

$a = 1/\sqrt{3}$. Therefore V is minimum for $a = 1/\sqrt{3}$.

59) Ans.(1)

Centroid of the three vectors is $(1/3)(i + 2j + k)$

60) Ans.(1)

$|\vec{AB}| = |\vec{OB} - \vec{OA}| = 9 \Rightarrow |-8i + (\lambda+3)j - k| = 9$

$$\Rightarrow \sqrt{64 + (\lambda+3)^2 + 1} = 9$$

$$\Rightarrow 65 + (\lambda+3)^2 = 81$$

$$\Rightarrow (\lambda+3)^2 = 16$$

$$\Rightarrow (\lambda+3) = \pm 4$$

$$\Rightarrow \lambda = 1 \text{ or } -7$$