ELEMENTS OF NUMBER THEORY & CONGRUENCES

Lagrange, Legendre and Gauss

Mathematics

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ELEMENTS OF NUMBER THEORY & CONGRUENCES

1) If $a \neq 0$, $b \neq 0 \in Z$ and a/b, b/a then

1) $a=b$
2) $a=1$
3) $b=1$
4) $a=\pm b$

Mathematics

Ans : is 4 known result.

If a/b $\Rightarrow$b=ma $\rightarrow$ (1) where m $\in$Z

& b/a $\Rightarrow$ a=bn $\rightarrow$ (2) where n $\in$Z

from (1) & (2), a=(am)n=a(mn)

$\Rightarrow$ mn=1, possible if m=1 & n=1

or m=-1 & n=-1. For the values of

n=1 &-1 then (2)$\rightarrow$a=$\pm$ b

Mathematics

2) 0 and 1 are

1) primes
2) composite numbers
3) neither prime nor composite
4) none of these

Mathematics

Ans : is 3

by defn. of prime & composite

numbers its implied

Mathematics

3) If (ab,c) = 1 & (a, c)=1 then (b, c)=

1) 1

2) c

3) b

4) none of the these

Mathematics

Ans : is 1

known result

$(a, c) = 1, \; (b, c) = 1 \Rightarrow (ab, c) = 1$

Mathematics

4) If p is prime number then p/ab $\Rightarrow$

1) p/a
2) p/b
3) p/a or p/b
4) none of the these

Mathematics

Ans : is 3. known result

$p/ab \Rightarrow p/a$ or p/b

Mathematics

5) 111..........1 (91 times) is

1) a composite number

2) a prime number

3) a surd

4) Irrational

Mathematics

Ans : is 1

since 91 = 7 x 13

$\underline{1111..1}_{\text{91 times}} = \underline{1111111}_{\text{7 times}} \cdot \underline{1111111}_{\text{7 times}}$ ----(13 factors)

& ∴ it is diviible by 1111111. (7 times)

∴ It is a composite number.

Mathematics

6) The number of positive divisors of 1400, including 1 and itself is

1) 18
2) 24
3) 22
4) 21

Mathematics

Ans : is 2

$$1400 = 2^3 \times 5^2 \times 7$$

$$\therefore\ T(1400) = (3+1)\,(2+1)(+1)$$

$$= 24$$

Mathematics

7) The sum of all positive divisors of 960 excluding 1 and itself is

1) 3047
2) 2180
3) 2087
4) 3087

Mathematics

Ans : is 3

$960 = 2^6 \times 3 \times 5$

$$S(960) = \left(\frac{2^{6+1}-1}{3-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right)\left(\frac{5^{1+1}-1}{5-1}\right)$$

$= 127 \times 4 \times 6 = 3048$

but 3048 – 960 – 1 = 2087.

Mathematics

8) If $(a+b)^3 \equiv x \pmod a$ then

1) $x=a^2$
2) $x=b^3$
3) $x=a^3$
4) $x=b^2$

Mathematics

Ans : is 2

$$(a+b)^3 = a^3+3a^2b + 3ab^2 + b^3$$

$$\Rightarrow (a+b)^3 - b^3 = a(a^2+3ab+3b^2)=ak$$

$$\Rightarrow a \mid [(a+b)^3 - b^3]$$

$$\therefore (a+b)^3 \equiv b^3 \pmod a$$

Mathematics

9) Which of the following statement is false ?

1) $98 \equiv -7 \pmod 3$

2) $67 \equiv 2 \pmod 5$

3) $123 \equiv -4 \pmod 7$

4) $240 \equiv 9 \pmod{11}$

Mathematics

Ans : is 3

123 + 4 = 127 is not a multiple of 7

Mathematics

10) If $100 \equiv x \pmod 7$, then the least positive value of x is

1) 1

2) 3

3) 4

4) 2

Mathematics

Ans : is 4

7 / (100 – x) when x = 2,

7 / 98

Mathematics

11) When 5^{20} is divided by 7 the remainder is

1) 1
2) 3
3) 4
4) 6

Mathematics

Ans : is 3

$5^3 = 125 \equiv -1 \pmod 7$

$\therefore (5^3)^6 \equiv (-1)^6 \pmod 7$

$5^{18}.5^2 \equiv 1.5^2 \pmod 7$

$\therefore 5^{20} \equiv 25 \pmod 7 \equiv 4 \pmod 7$

Mathematics

12) The last digit in 7^{291} is

1) 1
2) 3
3) 7
4) 9

Mathematics

Ans : is 2

$7^2 = 49 \equiv -1 \pmod{10}$

$\Rightarrow (7^2)^{145} \equiv (-1)^{145} \pmod{10}$

$7^{290} \equiv -1 \pmod{10}$

also $7 \equiv -3 \pmod{10}$

$\therefore\ 7^{190} x 7 \equiv (-1)(-3) \pmod{10}$

$\therefore\ 7^{291} \equiv 3 \pmod{10}$

Mathematics

13) The digit in the unit place of the number $183! + 3^{183}$ is

1) 7
2) 6
3) 3
4) 0

Mathematics

Ans : is 1

Unit place in 183 ! is 0 ($\because$ it is a factor of 10)

& $3^2 = 9 \equiv -1 \pmod{10}$

$(3^2)^{91} \equiv (-1)^{91} \pmod{10} = -1 \pmod{10}$

$\therefore\ 3^{182} \equiv -1 \pmod{10}$ also, $3 \equiv -7 \pmod{10}$

$\therefore\ 3^{182} . 3 \equiv (-1)(-7) \pmod{10}$

$\therefore\ 3^{183} \equiv 7 \pmod{10}$

Mathematics

14) If $-17 \equiv 3 \pmod{x}$, then x can take the value

1) 7

2) 3

3) 5

4) None of these

Mathematics

Ans : is 3

-17 – 3 = - 20 is divisible

by 5

Mathematics

15) The smallest positive divisor of a composite integer a (>1) does not exceed

1) a^2

2) $\sqrt[3]{a}$

3) a^3

4) $\sqrt{a}$

Mathematics

Ans : is 4

Known result

Mathematics

16) Which following linear congruences has no solution

1) $4x \equiv 1 \pmod{3}$

2) $3x \equiv 2 \pmod{6}$

3) $5x \equiv 3 \pmod{4}$

4) $2x \equiv 1 \pmod{3}$

Mathematics

Ans : is 2

Since (3, 6) = 3 & 3 does not divide 2

$\therefore$ No solution

Mathematics

17) The relation congruence modulo m is

1) Reflexive
2) Symmetric
3) Transitive only
4) All of these

Mathematics

Ans : is 4

Known result

$\because a \equiv b \pmod{m}$ is an equivalence relation

Mathematics

18) The least positive integer to which 79 x 101 x 125 is divided by 11 is

1) 5
2) 6
3) 4
4) 8

Mathematics

Ans : is 1

$79 \equiv 2 \pmod{11}$, $101 \equiv 2 \pmod{11}$

& $125 \equiv 4 \pmod{11}$ multiplying these,

$79 \times 101 \times 125 \equiv 2 \times 2 \times 4 \equiv 16 \pmod{11}$

but $16 \equiv 5 \pmod{11}$

$\therefore\ 79 \times 101 \times 125 \equiv 5 \pmod{11}$

Mathematics

19) If $p \equiv q \pmod{m}$ if and only if

1) $(p - q) / m$
2) $m/(p - q)$
3) m/p
4) m/q

Mathematics

Ans : is 2

by very defn. Of congruence

i.e. if $a \equiv b \pmod{m} \Rightarrow m/(a-b)$

Mathematics

20) When 2^{100} is divided by 11, the remainder is

1) 3
2) 5
3) 1
4) 2

Mathematics

Ans : is 3

$2^5 = 32 \equiv -1 \pmod{11}$

$\therefore (2^5)^{20} \equiv (-1)^{20} \pmod{11}$

$\therefore 2^{100} \equiv 1 \pmod{11}$

Mathematics

21) If $a \equiv b \pmod{m}$ and $(a, m) = 1$, then

1) $(a, b) = 1$
2) $(b, m) = 1$
3) $(b, m) = a$
4) $(a, b) = m$

Mathematics

Ans : is 2

Known result

$(a,m) = (b,m) = 1$

Mathematics

22) If $n \equiv 0 \pmod 4$ then $n^3 - n$ is divisible by

1) 6 but not 24
2) 12 but not 24
3) 24
4) 12 & 24

Mathematics

Ans : is 2

n is a multiple of 4

if n=4, $n^3 - n = 60$

$\therefore$ 12/60, 6/60 but 24 does not divided by 60

Thus 6 & 12 divide $n^3 - n$.

Mathematics

23) If $195 \equiv 35 \left(\mathrm{Mod}\ \overline{M+2}\right)$ then m =

1) 4
2) 5
3) 0
4) 7

Mathematics

Ans : is 3

$(m+2) / (195-35) \Rightarrow (m+2) / 160$

$\Rightarrow m+2 \geq 2$

$\Rightarrow m+2 = 2, 4, 5, 8$ - - - etc.

$\Rightarrow m= 0, 2, 3, 6$ etc.,

$\therefore$ (3) is the answer

Mathematics

24) If $2^8 \equiv (a+1) \pmod{7}$ is true then a is

1) 3
2) 4
3) 0
4) 5

Mathematics

Ans : is 1

$2^6 = 64 \equiv 1 \pmod{7}$

$2^6.2^2 \equiv 1.2^2 \pmod{7}$

$\therefore\ 2^8 \equiv 4 \pmod{7}$

$\Rightarrow a+1 = 4$ i.e., $(a=3)$

Mathematics

25) The unit digit in 13^{37} is

1) 5

2) 2

3) 6

4) 3

Mathematics

Ans : is 4

$$13^2 = 169 \equiv -1 \pmod{10}$$

$$(13^2)^{18} \equiv (-1)^{18} \pmod{10}$$

$$13^{36}.13 \equiv 1.13 \pmod{10}$$

$$\therefore\ 13^{37} \equiv 3 \pmod{10}$$

Mathematics

26) The number of incongruent solutions of $24x \equiv 8 \pmod{32}$ is

1) 2

2) 4

3) 6

4) 8

Mathematics

Ans : is 4

by thm.

(24, 32) = 8 & 8/8

$\therefore$ the number of incongruent

solutions = 8

Mathematics

27) The remainder when $3^{100} \times 2^{50}$ is divided by 5 is

1) 3
2) 4
3) 1
4) 2

Mathematics

Ans : is 2

$3^2=9 \equiv -1 \pmod 5 \Rightarrow (3^2)^{50} \equiv (-1)^{50} \pmod 5$

$\therefore 3^{100} \equiv 1 \pmod 5 \to (1)$

& $2^2=4 \equiv -1 \pmod 5 \Rightarrow (2^2)^{25} \equiv (-1)^{25} \pmod 5$

$\therefore 2^{50} \equiv -1 \pmod 5 \to (2)$

$(1) \times (2) \to 3^{100} \times 2^{50} \equiv 1 \times -1 \pmod 5 \equiv -1 \pmod 5$

but $-1 \equiv 4 \pmod 5$

$\therefore 3^{100} \times 2^{50} \equiv 4 \pmod 5$

Mathematics

28) If a and b are positive integers such that $a^2 - b^2$ is a prime number, then $a^2 - b^2$ is

1) a+b
2) a – b
3) ab
4) 1

Mathematics

Ans : is 1

$a^2 - b^2 = (a+b)(a-b)$ is a prime.

$\therefore (a+b)(a-b)$ is divisible by 1 or its self. But $a - b < a+b$ $\therefore$ $a-b=1$

$\therefore a^2 - b^2 = a+b$

Mathematics

29) Which of the following is a prime number ?

1) 370261
2) 1003
3) 73271
4) 667

Mathematics

Ans : is 1

17/1003, 11/73271 & 29/667.

but none of the

prime & less than 608

divides the first No.

Mathematics

30) Which of the following is false ?

1) An odd number is relatively prime to the next even number
2) $3x \equiv 4 \pmod{6}$ has solution
3) $ax \equiv bx \pmod{m}$; $x \neq 0 \Rightarrow a \equiv b \pmod{m}$
4) $a.x + b.y = d \Rightarrow (a, b) = d$

Mathematics

Ans : is 2

(3,6) = 3 but 3 does not divides 4

∴ no solution.

Remaining are all known results

Mathematics

31) For all positive values of p, q, r, and s, $\frac{(p^2+p+1)(q^2+q+1)(r^2+r+1)(s^2+s+1)}{pqrs}$ will not be less than

1) 81
2) 91
3) 101
4) 111

Mathematics

Ans : is 1

$$\frac{p^2 + p + 1}{p} = p + 1 + \frac{1}{p} \geq 3 \quad (\because p \text{ is +ve integer})$$

$|||^{ly}$ $$\frac{q^2 + q + 1}{q} = q + 1 + \frac{1}{q} \geq 3 \text{ etc.}$$

$\therefore$ **given expression is $\geq$ 3.3.3.3=81.**

$\therefore$ **expression cannot be less than 81.**

Mathematics

32) If $a+b)^n \equiv x \pmod{a}$, then (n is a +ve integer)

1) $x= a^2$
2) $x=a^n$
3) $x=b^n$
4) none of these

Mathematics

Ans : is 3

$(a+b)^n = a^n + {}^nc_1 a^{n-1}.b + \ldots + {}^nc_{n-1} ab^{-1} + b^n$

$\therefore (a+b)^n - b^n = a\,[a^{n-1} + {}^nc_1 a^{n-2}.b + \ldots + {}^nc_{n-1} b^{n-1}]$

$(a+b)^n - b^n = ak$ where $k \in Z$.

$\therefore a/[(a+b)^n - b^n]$

$\Rightarrow (a+b)^n \equiv b^n \pmod a$

$\therefore x = b^n$

Mathematics

33) If 27= 189m + 24n then m & n are

1) unique
2) not unique
3) prime numbers
4) none of these

Mathematics

Ans : is 2

If (a,b) = d $\Rightarrow$ d = ax + by

where x, y $\in$ Z. Here x, y are not unique.

Mathematics

34) If $2x \equiv 3 \pmod 7$, then the values of x such that $9 \leq x \leq 30$ are

1) 12, 19, 26
2) 11, 18, 25
3) 10, 17,24
4) None of these

Mathematics

Ans : is 1

The soln. is $x \equiv 5 \pmod 7$

$\therefore$ Soln. set is { ….. 2, 5, 12, 19, 26, 33, …. }

$\therefore$ required values of x are 12, 19, 26.

Mathematics

35) If p is a prime number and P is the product of all prime numbers less than or equal to p_1 then

1) P – 1 is a prime
2) P + 1 is not a prime number
3) P + 1 is a prime number
4) P + 1 is a composite number

Mathematics

Ans : is 3

Known result while proving

the thm. The primes are infinite.

Mathematics

36) $4x + 9 \equiv 3 \pmod{5}$ can be written as

1) $x \equiv 5 \pmod{6}$
2) $x \equiv 3 \pmod{15}$
3) $x \equiv 6 \pmod{15}$
4) None of these

Mathematics

Ans : is 3

when x=6, 4.6+9 = 33 $\equiv$ 3 (mod 5)

it satisfies the given congruence.

Hence (3) is right answer

Mathematics

37) If $(3-x) \equiv (2x-5) \pmod 4$, then one of the values of x is

1) 3
2) 4
3) 18
4) 5

Mathematics

Ans : is 2

3-x-2x+5 = -3x+8 is divisible by 4

when x=4, -3 (4)+8 = -4

is divisible by 4.

Mathematics

38) The remainder when 64x65x66 is divided by 67 is

1) 60
2) 61
3) 62
4) 63

Mathematics

Ans : is 2

$$64 \times 65 \times 66 \equiv (-3)(-2)(-1) \pmod{67}$$

$$\equiv -6 \pmod{67}$$

$$\equiv 61 \pmod{67}$$

Mathematics

GROUPS

Lagrange, Legendre and Gauss

Mathematics

GROUP

1) If x,y,z are three elements of a group and then $(xy^{-1}z)^{-1}$=

1) $x^{-1}y^{-1}z^{-1}$
2) $x^{-1}yz$
3) $z^{-1}yx^{-1}$
4) $(xy^{-1}z)^{-1}$

Mathematics

Ans : is 3

since $(a \star b)^{-1} = b^{-1} \star a^{-1}$.

Question is just extension of this property.

Mathematics

2) If $a * b = \sqrt{a} + \sqrt{b}$, then $*$ is a binary operation on

1) R
2) Q^+
3) R_o
4) R^+

Mathematics

Ans : is 4

if a = -1, b = 3 then

$$a \star b = \sqrt{-1} + \sqrt{3} \in C$$

Mathematics

3) The identity element of $a \ast b = a^{b-1}$ is

1) 1

2) 0

3) 2

4) – 1

Mathematics

Ans : is 3

$a * e = a \Rightarrow a^{e-1} = a$

$\Rightarrow e - 1 = 1 \Rightarrow e = 2$

Mathematics

4) In the group of rational numbers under a binary operation $\ast$ defined by $a \ast b = a+b-1$ then identity element is

1) 1

2) 0

3) 2

4) -1

Mathematics

Ans : is 1

$a \star e = a \Rightarrow a+e-1=a$

$\therefore \; e - 1 = 0 \Rightarrow e=1$

Mathematics

5) The set G={ -3, -2, -1, 0, 1, 2, 3} w.r.t. addition does not form a group since.

1) The closure axiom is not satisfied
2) The associative axiom is not satisfied
3) The commutative axiom is not satisfied
4) Identity axiom is not satisfied

Mathematics

Ans : is 1

since 2, $3 \in G$ but $2+3=5 \notin G$

Mathematics

6) If $a \ast b = 2a - 3b$ on the set of integers. Then $\ast$ is

1) Associative but not commutative
2) Associative and commutative
3) A binary operation
4) Commutative but not associative

Mathematics

Ans : is 3

$\forall$ a, b$\in$Z, a$\star$b=2a-3b $\in$Z

(i.e., if a =1, b=-2 then

2 .1 -3 (-2) = 2+6= 8$\in$Z)

Mathematics

7) In the multiplicative of cube roots of unity the inverse of w^{99} is

1) w

2) 1

3) w^2

4) Does not exist.

Mathematics

Ans : is 2

$W^3=1$

$\therefore (w^3)^{33} = 1$

Mathematics

8) The incorrect statement is

1) In (G, .) $ab=ac \Rightarrow b=c, \forall\ a, b, c \in G$
2) Cube roots of unity form an abelian group under addition
3) In a abelian group $(ab)^3=a^3b^3, \forall a, b \in G$
4) In a group of even order, there exists atleast two elements with their own inverse.

Mathematics

Ans : is 2

Cube roots of unity; 1, w, w^2

form an abelian group

under multiplication

Mathematics

9) If H & K are two subgroups of a group G, then identify the correct statement

1) $H \cap K$ is a sub group
2) $H \cup K$ is a sub group
3) Neither $H \cup K$ nor $H \cap K$ is sub group
4) Nothing can be said about $H \cup K$ and $H \cap K$

Mathematics

Ans : is 1

Let H= {0, 2, 4}, K={0,3} are sub groups of G={0, 1, 2, 3, 4, 5} under $+_6$

i.e., $H \cup K$ = {0, 2, 3, 4} is not closed

i.e., $2+3=5 \notin H \cup K$

Mathematics

10) In the group G= {e, a, b} of order 3, a^5b^4 is

1) 3
2) ab
3) a
4) b

Mathematics

Ans : is 3

$ab=e \Rightarrow (ab)^4 = e$

i.e. $a\,(a^4b^4) = ae$

$\Rightarrow a^5b^4=a$

Mathematics

11) In a group (G, $\star$), $a \star x = b$ where a, b $\in$ G has

1) Unique solution
2) No solution
3) More than one solution
4) Infinite number of solution

Mathematics

Ans : is 1

$$a \ast x = b \Rightarrow a^{-1} \ast (a \ast x) = a^{-1} \ast b$$

$$(a^{-1} \ast a) \ast x = a^{-1} \ast b \Rightarrow x = a^{-1} \ast b$$

Mathematics

12) The set of (non singular) matrices of order 2 x 2 over z under matrix multiplication is

1) Group
2) Semi group
3) Abelian group
4) Non-abelian group

Mathematics

Ans : is 2

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in M$$

$$\text{When } A = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}, |A| = 2 \text{ but } A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix}$$

but $\frac{1}{2} \notin Z$

Mathematics

13) Which of the following is a subgroup of G={0, 1, 2, 3, 4, 5} under addition modulo 6

1) {0, 2}
2) {0, 1}
3) {0, 4}
4) {0, 3}

Mathematics

Ans : is 4

$2+_6 2=4 \notin \{0,2\}$ etc.,

but $3+_6 3=0$

Mathematics

14) The set of integers is

1) Finite group
2) Additive group
3) Multiplicative group
4) None of these

Mathematics

Ans : is 2

Mathematics

15) The set of all integers is not a group under multiplication because

1) Closure property fails
2) Associative law does not hold good
3) There is no identity element
4) There is no inverse

Mathematics

Ans : is 4

Inverse 0 does not exists

(also $2 \in z$ but $2^{-1} = \frac{1}{2} \notin z$)

Mathematics

16) A subset H of a group (G, $\star$) is a subgroup of G iff

1) a, b$\in$H $\Rightarrow$ a $\star$ b$\in$H

2) a$\in$H $\Rightarrow$ $a^{-1}\in$H

3) a, b$\in$H $\Rightarrow$ a $\star$ $b^{-1}\in$H

4) H contains identity off G.

Mathematics

Ans : is 3

By thm.

Mathematics

17) $Z_n = \{0, 1, 2, - - - -, (n-1)\}$ fails to be a group under multiplication modulo n because

1) Closure property fails
2) Closure holds but not associativity
3) There is no identity
4) There is no inverse for an element of the set

Mathematics

Ans : is 4

at least for one element '0'

has no inverse in Z_n.

Mathematics

18) $G = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} : x \neq 0 \ \& \ x \in R \right\}$ is an abelian group under matrix multiplication. Then the identity element is

1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

3) $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

4) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

Mathematics

KEA

Ans : is 3

$$A(x) = \begin{bmatrix} x & x \\ x & x \end{bmatrix}, A(e) = \begin{bmatrix} e & e \\ e & e \end{bmatrix} \text{ then}$$

$$A(x).A(e) = A(x) \text{ then}$$

$$\begin{bmatrix} 2xe & 2xe \\ 2xe & 2xe \end{bmatrix} = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \Rightarrow 2xe = x$$

$$\Rightarrow e = ½ \therefore A(e) = \begin{bmatrix} ½ & ½ \\ ½ & ½ \end{bmatrix}$$

Mathematics

19) In the group G = {3, 6, 9, 12} under $\times_{15}$, the identity is

1) 3

2) 6

3) 9

4) 12

Mathematics

Ans : is 2

Since $3 \times_{15} 6 = 3$, $6 \times_{15} 6 = 6$

$9 \times_{15} 6 = 9$ etc.,

Mathematics

20) The set of all 2 x 2 matrices over the real numbers is not a group under matrix multiplication because

1) Inverse law is not satisfied
2) Associative law is not satisfied
3) Identity element does not exist
4) Closure law is not satisfied

Mathematics

Ans : is 1

If A is a singular matrix

of 2 x 2 order matrix then

A^{-1} does not exist.

Mathematics

21) $(Z, \ast)$ is a group with $a \ast b = a+b+1, \ \forall \ a, b \in Z$. The inverse of a is

1) A+2

2) – a+2

3) – a –2

4) a – 2

Mathematics

Ans : is 3

$a \star e = a \Rightarrow a + e + 1 = a \Rightarrow e = -1$

$a \star a^{-1} = e \Rightarrow a + a^{-1} + 1 = -1$

$\Rightarrow a^{-1} = -2 - a$

Mathematics

22) The four matrices $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$, $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$, $\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$ under multiplication form is

1) a group
2) a semi group
3) an abelian group
4) infinite group

Mathematics

Ans : is 3

Taking them as I, A, B, C

then AB=C, BC=A, etc., & A.I=A etc.

Also, A.A=I $\Rightarrow$ A^{-1}=A |||^{ly} B^{-1}=B,

C^{-1}=C also AB=BA

Mathematics

23) In the group (G, $\star$), $a \star b = \frac{ab}{5}$ where $\forall$ a, b $\in$ G. The identity and inverse of 8 are respectively.

1) $5 \ \& \ \frac{5}{3}$

2) $5 \ \& \ \frac{25}{8}$

3) $5 \ \& \ \frac{8}{25}$

4) $5 \ \& \ \frac{3}{5}$

Mathematics

Ans : is 2

$a \star e = a \Rightarrow ae/5 = a \Rightarrow e = 5$

& $a \star a^{-1} = e \Rightarrow \dfrac{aa^{-1}}{5} = 5 \Rightarrow a^{-1} = \dfrac{25}{a}$

$\therefore 8^{-1} = \dfrac{25}{8}$

Mathematics

24) The proper subgroups of the group G = {0, 1, 2, 3, 4, 5} under addition modulo 6 are

1) {0, 3} and {0, 2, 4}
2) {0, 1, 3} and {0, 1, 4}
3) {0, 1} and {3, 4, 5}
4) {0} and {0, 1, 2, 3, 4, 5}

Mathematics

Ans : is 1

Since 0(G)=6 & 6=2 x 3

$\therefore$ It has proper subgroups of orders 2 & 3

In (1) $3+_6 3=0$ & $2+_6 2=4$, $4+_6 2=0$

$4+_6 4=2$ all in the sets

Mathematics

25) In the group G = {1, 3, 7, 9} under multiplication modulo 10, the value of $(3 \times_{10} 7^{-1})^{-1}$ is

1) 5

2) 3

3) 7

4) 9

Mathematics

Ans : is 4

$e=1$

$7 X_{10} 3=1 \Rightarrow 7^{-1}=3$

$\therefore 3 X_{10} 3=9$

Mathematics

26) The incorrect statement is

1) The identity element in a group is unique
2) In a group of even order, there exists an element $a \neq e$ such that $a^2 = e$.
3) The cube roots of unity are , $1, \frac{1-i\sqrt{3}}{2}, \frac{1+i\sqrt{3}}{2}$
4) In an abelian group $(ab)^2 = a^2b^2, \forall\ a, b \in G$.

Mathematics

Ans : is 3

Cube roots of unity are

$$1, \frac{-1 \pm i\sqrt{3}}{2}, \quad \text{where}$$

$$1, w = \frac{-1 + i\sqrt{3}}{2}, w^2 = \frac{-1 - i\sqrt{3}}{2}$$

Mathematics

27) In the multiplicative group of fourth roots of unity the inverse of i^{103} is

1) 1

2) – 1

3) i

4) – i

Mathematics

Ans : is 3

$e = 1$

$i^{103} = i^{100} \cdot i^3 = (i^4)^{25} \cdot (i^2).i$

$= 1. (-1) .i = -i$

$\therefore$ inverse of $-i$ is i.

Mathematics

28) Let $Q_1 = Q - \{1\}$ be the set of all rationals except 1 and $\star$ is defined as $a \star b = a + b - ab \;\forall\; a, b \in Q_1$. The inverse of 2 is

1) 2

2) 1

3) 0

4) – 2

Mathematics

Ans : is 1

$a \star e = a \Rightarrow a + e - ae = a$

$\Rightarrow e(1-a) = 0 \Rightarrow e = 0 \ (\because a \neq 1 \notin Q_1)$

$\& \ a \star a^{-1} = e \Rightarrow a + a^{-1} - aa^{-1} = 0$

$\Rightarrow a^{-1}(1-a) = -a \Rightarrow a^{-1} = \dfrac{-a}{1-a}$

$(\because 1-a \neq 0)$

$\therefore 2^{-1} = \dfrac{-2}{1-2} \Rightarrow 2^{-1} = 2$

Mathematics

29) In the group $\{Z_6, + (\text{mod } 6)\}$, $2+4^{-1} + 3^{-1}$ is equal to

1) 2
2) 1
3) 4
4) 3

Mathematics

Ans : is 2

e=0

$\therefore\ 2 +_6 4^{-1} +_6 3^{-1} = 2 +_6 2 +_6 3 = 1$

Mathematics

30) Every group of order 7 is

1) Not abelian
2) Not cyclic
3) Cyclic
4) None of these

Mathematics

Ans : is 3

Every group of prime

order is cyclic

7 is prime

Mathematics

31) If g = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and h = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$ are two permutations in group S_4, then (h x g) (2) =

1) 2
2) 1
3) 3
4) 4

Mathematics

Ans : is 2

$(h \times g)2 = h[g(2)] = h(3) = 1$

Mathematics

32) If $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ then g^{-1}

1) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

2) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$

3) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$

4) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

Mathematics

Ans : is 1

$$g^{-1} = \begin{pmatrix} 3 & 4 & 1 & 2 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$$\therefore\ g^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$$

Mathematics

33) In the group {1, 2, 3, 4, 5, 6} under multiplication modulo 7, 5x=4 has the solution x =

1) 0.8
2) 2
3) 3
4) 5

Mathematics

Ans : is 4

(e=1)

$5x_7 3=1 \Rightarrow 5^{-1} = 3$

$\therefore\ 5x=4 \Rightarrow x= 5^{-1}x_7 4 = 3x_7 4=5$

Mathematics

34) In the group G={2, 4, 6, 8} under X_{10}, the inverse of 4 is

1) 6

2) 8

3) 4

4) 2

Mathematics

Ans : is 3

Here e=6 since $4x_{10}6=4$ etc.

$\therefore\ 4x_{10}4=6 \Rightarrow 4^{-1}=4$

Mathematics

35) The Set { –1 , 0, 1} is not a group w.r.t. addition because it does not satisfy

1) Closure property
2) Associative law
3) Existence of identity
4) Existence of inverse

Mathematics

Ans : is 1

$1+1=2 \notin$ the set

Mathematics

36) If every element of a group G is its own inverse, then G is

1) Finite
2) Infinite
3) Cyclic
4) Abelian

Mathematics

Ans : is 4

since $a = a^{-1}$, $b = b^{-1}$ $\forall$ $a, b \in G$

Now $(ab)^{-1} = ab$ (by hypothesis)

$\Rightarrow b^{-1}a^{-1} = ab$, by property

$\Rightarrow ba = ab$

$\therefore$ G is abelian

Mathematics

37) If a, b, c, are three elements of a group $(G, \star)$, and $(a\star b)\star x=c$, then x=

1) $c\star(a^{-1}\star b^{-1})$
2) $c\star(b^{-1}\star a^{-1})$
3) $(b^{-1}\star c^{-1})\star c$
4) $(a^{-1}\star b^{-1})\star c$

Mathematics

Ans : is 3

$$(a \star b)^{-1} \star (a \star b) \star x = (a \star b)^{-1} \star c$$

$$e \star x = (b^{-1} \star a^{-1}) \star c$$

Mathematics

38) If $\{ z_7, x_7\}$ is a group, then the inverse of 6 is

1) 6
2) 4
3) 1
4) 3

Mathematics

Ans : is 1

since $6 \times_7 6 = 36 \equiv 1 \pmod 7$

where e = 1

$\therefore\ 6^{-1} = 6$

Mathematics