Co-ordinate Geometry

1.Reflection of (-4,3) on X-axis is …..

a.(4,-3)

b.(-4,-3)

c.(4,3)

d.None of these

Answer: b.Change the sign of ordinate

2.The slope & intercept of x-y+1=0 is…

a)1,1

b)1,-1

c)-1,1

d)-1,-1

Answer: a

Solution: x-y+1=0 => y=x+1. (y=mx+c),

i.e. m=c=1.

3.The equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents as circle if …..

a)h=0 and a≠b

b)h≠0 and a=b

c)h≠0 and a≠b

d)h=0 and a=b

Answer is d. It represents the standard formula of circle

4.The number of common tangents to the circles $x^2+y^2=4$ & $x^2+y^2-6x-8y=24$ is….

a)2

b)1

c)3

d)None of these

Answer is b.

5)The lines 2x-3y=5 & 3x-4y=7 are diameters of the circles having area as 154sq.Units, then the equation of the circle is ….

a) $x^2+y^2-2x-2y+47=0$

b) $x^2+y^2+2x-2y-47=0$

c) $x^2+y^2-2x+2y-47=0$

d)None of these

Answer is c.

Solution: solve the given equation (1)&(2)

2x-3=5..(1) 3x-4y=7...(2) we get x=1,y=-1,

i.e., the centre of the circle is(1,-1) area of the circle $A= \pi r^2$

$r^2=154/\pi=154/(22/7)=(154)7/22=49$

Hence the equation of the circle is

$(x-h)^2_+ (y-k)^2 =r^2$ i.e., $(x-1)^2+(y+1)^2=49$

i.e., $x^2+y^2-2x+2y-47=0$

6.The intercept on the line y=x by the circle $x^2+y^2-4x=0$ is A&B. Find the equation of the circle on AB as diameter.

a)$x^2+y^2-2x-2y=0$

b)$x^2+y^2+2x+2y=0$

c)$x^2+y^2+2x-2y=0$

d)$x^2+y^2-2x+2y=0$

Answer is a.

Solution: Given equation of circle $x^2+y^2-4x=0$…(1) is

Given line is y=x..(2), put y=x in (1), $x^2+y^2-4x=0$ $2x^2-4x=0$ => $2x(x-2)=0$ therefore x=0, x=2

When x=0, y=0, when x=2, y=2, A(0,0),B(2,2), (x_1, y_1) (x_2, y_2)

equation of the circle on AB as diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$\Rightarrow (x-0)(x-2)+(y-0)(y-2)=0$

$\Rightarrow x^2-2x+y^2-2y=0$

I.e. $x^2+y^2-2x-2y=0$ is the equation of the circle on AB

7.The equation of the circle is $x^2+y^2+4x-4y+4=0$ which makes equal intercepts on the +ve co-ordinate axes.

Then the equation of tangent is…

a)$x-y+2\sqrt{2}=0$

b)$x-y-2\sqrt{2}=0$

c) $x+y-2\sqrt{2}=0$

d)None of these,

Answer is c.

Solution: Equation of the tangent is (x/a)+(y/b)=1--(i)Put b=a in(i),x+y=a--(ii)

Centre(-2,2), radius=r=$\sqrt{2}$+4-4=2

Length of perpendicular from (-2,2) on (ii)=radius

I,e.{ [1(-2)+1(2)-a]$/\sqrt{1^2+1^2}$ }=2

{(-2+2-a)/ $\sqrt{2}$}=2 =>a=2$\sqrt{2}$

In equation (ii) x+y=2$\sqrt{2}$ or x+y-2$\sqrt{2}$=0

8.If $x+y=p$ is normal is $y^2 = 16x$ then p is .....

a)10

b)4

c)3

d)12

Answer is d.

Solution: if $y=mx+c$ ….(1) is a normal to

$y^2 = 4ax$ if $c=-2am-am^3$ ...(2)

$4a=16$ $a=4$, given $x+y=p$

$\Rightarrow y=-x+p$ therefore $m=-1, c=p$

Now equation (2) $\Rightarrow p=-2(4)(1)^3$

$=8+4$

$=12$

9.If the parabola $y=x^2-5x+6$ at the points (2,0)&(3,0). Then the angle between the tangents to the parabola is….

a) π/2

b) π/4

c) π

d)None of these

Answer is a.

Solution:If $y=x^2-5x+6$

$dy/dx=2x-5$

$[dy/dx](2,0)=2.2-5=4-5=-1=m_1$

$dy/dx=2x-5$

$[dy/dx](3,0)=2.3-5=6-5=1=m_2$

$\Rightarrow m_1m_2=-1$

=> Angle between the tangents= $\pi/2$

10.If e=1/2 & one of the directrix is x=4, then the equation of the ellipse is ….

a) $x^2/9+y^2/4 =1$

b) $x^2/8+y^2/9 =1$

c) $x^2/4+y^2/3 =1$

d)None of these

Answer is c.

Solution: given $e=1/2$ & $x=a/e$

$\Rightarrow 4=a/(1/2)$

$\Rightarrow 4=2a \Rightarrow a=2 \Rightarrow a^2=4$

Now $b^2 = a^2(1-e^2) = 4(1-1/4)=4(3/4)$,

$b^2=3$

Therefore equation of the ellipse is

$x^2/4+y^2/3=1$

11.The hypothesis $x^2/\cos^2\alpha - y^2/\sin^2\alpha = 1$ then abscissa of foci…., when α varies,

a)(1,0)

b)(-1,0)

c)(0,0)

d)(±1,0)

Answer is d.

Solution: we know that $b^2 = a^2(e^2 - 1)$

when $a=\cos\alpha$ $b=\sin\alpha$

$\sin^2\alpha = \cos^2\alpha(e^2-1)$

By dividing $\cos^2\alpha$, $\tan^2\alpha=e^2-1$

$\Rightarrow 1+\tan^2\alpha=e^2$

$\Rightarrow \mathrm{Sec}^2\alpha=e^2$

$e=\sec\alpha$

coordinate of foci are $(\pm ae,0)$

$\Rightarrow(\pm \cos\alpha.\ \sec\alpha,o)$

$\Rightarrow(\pm\cos\alpha.1/\cos\alpha,o)$

$\Rightarrow(\pm1,0)$

12.The locus of a point p(α,ß) moving condition that line $y=\alpha x+ß$ is a tangent to the hyperbola $(x^2/a^2)-(y^2/b^2)=1$is a….

a)Hyperbola

b)Parabola

c)Ellipse

d) None of these

Answer is d.

Solution:If $y=\alpha x+ß$ touches

$(x^2/a^2)-(y^2/b^2)=1$

If $ß^2=a^2\alpha^2 -b^2$

therefore locus of $(\alpha, ß)$ is

$y^2=a^2x^2-b^2$

$\Rightarrow b^2 =a^2x^2-y^2$

13.If a line makes an angle of with the +ve direction to the x-axis and y-axis, then the angle that line makes with the +ve direction to the z-axis is…

a) $\pi/3$

b) $\pi/2$

c) $\pi/4$

d) $\pi/6$, Answer is b.

Solution: By using $\cos^2 p+\cos^2 q+\cos^2 r=1$

$\cos^2 \pi/4 +\cos^2 \pi/4 +\cos^2 r =1$

$1/2+1/2+\cos^2 r=1-1=0$

$\cos r=0$

$\Rightarrow r= \pi/2$

14. A focus of an ellipse is at the origin, the directrix is the line x+4 & the eccentricity is ½. Then the length of the semi-major axis is….

a)8/3

b)2/3

c)4/3

d)5/3,

Answer is a.

Solution: Major axis along x-axis

$(a/e)-ae=4$

$a\{1/(1/2)\}-1/2=4$

$a\{2-1/2\}=4$

$a(3/2)=4$

$3a=8,\ \ a=8/3$

15.The point diametrically opposite to the point p(α, ß) on the circle $x^2+y^2+2x+4y-3=0$ is….

a)(-3,4)

b)(3,-4)

c)(-3,-4)

d)(3,4), Answer is c.

Solution: centre of circle is (-1,-2)

Let (α, ß) is the required point

$(\alpha+1)/2=-1$, $(ß+0)/2=-2$

$\alpha+1=-2$ $ß=-4$

$\alpha=-3$

therefore (α, ß)=(-3,-4)

16.Length of the chord of the circle $x^2+y^2-6x+4y+5=0$ is intercepted by x-axis is…

a)4units

b)2 units

c)0

d)none of these

Answer is a.

Solution: Equation of the circle is

$x^2+y^2-6x+4y+5=0$

$g=-3,\ f=0, c=5$

$x\text{-intercepted}=2\sqrt{g^2-c}$

$=2\sqrt{3^2-5}$

$=2\sqrt{4}$

$=2(2)=4$ units

17.If the vertex of the parabola $Y=x^2-8x+c$ lies on x-axis then the value of c is….

a)16

b)4

c)-16

d) None of these

Answer is a.

Solution: $x^2-8x=y-c \Rightarrow (x-4)^2-16=y-c$

$(x-4)^2=y-c+16 \Rightarrow (x-4)^2=[y-(c+16)]$

Therefore vertex v(4,c-16)

It is given that lies on x-axis

hence y co-ordinate is zero

i.e., $c-16=0 \Rightarrow c=16$

18.If $x+y=k$ is normal to $y^2=12x$, then k is….

a)3

b)-3

c)9

d)none of these

Answer is c.

Solution: $y=mx+c$ is normal to $y^2=4ax$,

if $c=-2am-am^3$---(i)

When $4a=12$, $y=-x+k$ => $m=-1$, $c=k$

$a=3$, (i)=> $k=-2(3)(-1)^2-3(-1)^3$

$k=6+3=9$

$k=9=c$

19.Equation of $x^2+y^2-4x+6y+8=0$ from (-5,-4)is…

a)3x+y+14=0

b)x+2y-3=0

c)2x-2y+6=0

d)None of these

Answer is a.

Solution: Equation of tangent is

$x\,x_1 + yy_1 + g(x+x_1)+f(y+y_1)+c=0\ldots\ldots(i)$

$g=2, f=3,\ c=8, x_1=-5, y_1=-4$

In(1) $x(-5)+y(-4)+2(x-5)+3(y-4)+8=0$

$-5x-4y+2x-10+3y-12+8=0$

$-3x-y-14=0$

$3x+y+14=0$ is the equation of the tangent.

20.The given equation of the circle is $x^2+y^2-4x-3y+4=0$ Then it touches…

a)x-axis

b)y-axis

c)co-ordinate axes

d)none of these

Answer is a.

Solution: Where g=2, c=4,

if $g^2 = c$ then $2^2 = 4$

$4 = 4$

Therefore circle touch x-axis

21.If y= x+c may be tangent to the parabola $y^2=12x$ then the co-ordinates of the point of contact is…..

a)(1,2)

b)(3,4)

c)(3,6)

d)None of these

Answer is c.

Solution: $y^2=12x$

$y^2=4ax$ therefore $a=3$

If $y=x+c$ where $c=a/m=3/1=3$

Point of contact $(a/m^2 , 2a/m)$

I.e. $(3/1, 2.3/1)=(3,6)$

22.If the latus rectum is 4 & distance between foci is $2\sqrt{15}$.Then the equation of ellipse is….

(The standard form of the ellipse is $x^2/a^2 + y^2/b^2 = 1 a>b$)

a) $x^2/25 + y^2/10 = 1$ b) $x^2/5 + y^2/10 = 1$

c) $x^2/10 + y^2/25 = 1$ d) None of these

Answer is a.

Soution:$2b^2/a=4 \Rightarrow b^2=2a$.

$2ae=2\sqrt{15} \Rightarrow ae=\sqrt{15}$

We know that $b^2=a^2-a^2e^2$

$2a=a^2-15 \Rightarrow a^2-2a-15=0$

Therefore $a=5, a=-3$

in $b^2 = 2a=2.5=10$ neglect the value of $a=-3$.Hence equation of ellipse is

$x^2/25+y^2/10=1$

23.The distance between foci is 8 & distance between directrices is 9/2, the equation of hyperbola is….

a) $x^2/36+y^2/45=1$

b) $x^2/9-y^2/7=1$

c) $x^2/45-y^2/36=1$

d) None of these

Answer is b.

Solution: if 2ae=8, ae=4….(i)

And 2a/e=(9/2)

Therefore a=9/4(e)….(ii)

From(ii) in (i) I,e. 9/4(e).e=4 =>e^2=16/9 => e=4/3 In (i) a.4/3=4 => a=3 =>a^2=9

b^2 =a^2 (e^2 -1) => b^2 =9(16-9)/9=7 the equation of hyperbola is x^2/9-y^2/7=1

24. The eccentricity of a hyperbola is $\sqrt{3}$ then eccentricity of its conjugate is…..

a)$2/\sqrt{3}$

b)$\sqrt{3}/\sqrt{2}$

c) $\sqrt{3}/2$

d)$3/\sqrt{2}$

Answer is b.

Solution: $e_1=\sqrt{3}$, $e_2=?$

By using formula $1/(e_1)^2 + 1/(e_2)^2 = 1$

$\Rightarrow 1/3+1/(e_2)^2 = 1$

$\Rightarrow 1/(e_2)^2 = 1-1/3=2/3$

then $e_2= \sqrt{3}/\sqrt{2}$

25.In a standard equation of a hyperbola with the centre of the origin SS'=16 & e=$\sqrt{2}$ then the equation is….

a) $x^2-y^2 = 32$

b) $x^2-y^2 = 16$

c) $y^2-x^2 = 16$

d) $y^2-x^2 = 32$

Answer is a.

Solution: 2ae=16, ae=8

a^2e^2 =64….(i)

given e=$\sqrt{2}$ => e^2=2.

In(i) substitute the value of e^2

we get $a^2=32=b^2$

then the standard equation of hyperbola is x^2-y^2 =32