CALCULAS

Vikasana - CET 2012

CALCULAS
PART-A
MULTIPLE CHOICE QNS

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$$1. \text{If } y = \sqrt{\sin x}, \text{ then}$$

$$\frac{dy}{dx} =$$

a) $\dfrac{\cos x}{2\sqrt{\sin x}}$

b) $\dfrac{\sin x}{2\sqrt{\sin x}}$

c) $\dfrac{\cos x}{\sqrt{\sin x}}$

d) $\dfrac{2\cos x}{\sqrt{\sin x}}$

SOLUTION: ANSWER ©

$$2. If.\ f(x) = x(\sqrt{x} - \sqrt{x+1})$$

$$then.\ f'(0) =$$

$a)\,0$

$b)\,1$

$c)\,-1$

$d)\,Infinity$

$$Solution: f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{x} - \sqrt{x+1})}{x} = -1$$

$$Ans: (c)$$

$$3.\ \frac{d}{dx}\left[\sin^2 \cot^{-1} \sqrt{\frac{1+x}{1-x}}\right] =$$

$a)\, 0$

$b)\, 1/2$

$c)\, -1/2$

$d)\, -1$

$$Solution: put.\, x=\cos\theta \Rightarrow \cot^{-1}\sqrt{\frac{1+x}{1-x}}=\frac{\theta}{2}$$

$$\Rightarrow y=\sin^2\frac{\theta}{2}=\frac{1}{2}(1-\cos\theta)$$

$$\Rightarrow y=\frac{1}{2}(1-x)\Rightarrow\frac{dy}{dx}=\frac{-1}{2}$$

$$Ans:(c)$$

$$4.\ If\ .x = e^{y+e^{y+\cdots\infty}} \quad then.\frac{dy}{dx} =$$

$$a)\ \frac{1-x}{x}$$

$$b)\ \frac{x}{1-x}$$

$$c)\ \frac{1+x}{x}$$

$$d)\ \frac{x}{1+x}$$

$$Solution: x = e^{y+x} \Rightarrow \log x = y + x$$

$$\Rightarrow \frac{1}{x} = y' + 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$$

$$Ans: (a)$$

$$5. If\ x = \theta\cos\theta + \sin\theta \ \&\ y = \cos\theta - \theta\sin\theta, then$$

$$\frac{dy}{dx}\ at\ \theta = \frac{\pi}{2}$$

$a) -\frac{\pi}{2}$

$b) \frac{2}{\pi}$

$c) \frac{\pi}{4}$

$d) \frac{4}{\pi}$

$$Solution: x=\theta\cos\theta+\sin\theta; y=\cos\theta-\theta\sin\theta$$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{-\sin\theta-\theta\cos\theta}{-\theta\sin\theta+\cos\theta+\cos\theta}$$

$$\Rightarrow\left[\frac{dy}{dx}\right]_{\theta=\pi/2}=\frac{-1-1}{-\pi/2}=\frac{4}{\pi}$$

$$Ans: d)$$

6. The derivative of $\sin^2 x$ w.r.t. $(\log x)^2 =$

a) $\dfrac{x.\sin x.\cos x}{\log x}$

b) $\dfrac{2\sin x.\cos x}{(\log x)^2}$

c) $\dfrac{\sin^2 x}{2\log x}$

d) $x.\log x$

$$Solution: u=\sin^2 x, v=(\log x)^2$$

$$\frac{du}{dv}=\frac{2\sin x.\cos x}{2.\log x.\left(\frac{1}{x}\right)}=\frac{x.\sin x.\cos x}{\log x}$$

$$Ans: a)$$

$$7.\ If.\ y = \left(x^x\right)^x, then \left(\frac{dy}{dx}\right)_{x=1} =$$

$a) 1 + \log 2$

$b) 1$

$c) -1$

$d) 1 - \log 2$

$$Solution: y=\left(x^{x}\right)^{x} \Rightarrow \log y = x\log(x^{x})$$

$$\Rightarrow \log y = x^{2}.\log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x^{2}.\frac{1}{x} + 2x.\log x$$

$$\frac{dy}{dx} = xy(1+2\log x)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = 1$$

$$Ans: b)$$

$8. If . f(x) = x^n . then.the.value.of$

$$f(1) - \frac{f'(1)}{1\dagger} + \frac{f''(1)}{2\dagger} - \ldots\ldots\ldots\ldots + (-1)^n \frac{f^{(n)}(1)}{n\dagger}$$

is

$a)1$

$b)-1$

$c)0$

$d)\infty$

$$Solution: f(1)=1, f'(1)=n, f''(1)=n(n-1)$$

$$f'''(1)=n(n-1)(n-2)$$

$$G.E.=1-\frac{n}{1!}+\frac{n(n-1)}{2!}-\frac{n(n-2)(n-3)}{3!}+........+(-1)^n\frac{n!}{n!}$$

$$=nc_o-nc_1+nc_2-nc_3+..........+(-1)^n nc_n=(1-1)^n=0$$

$$Ans:(c)$$

$$9.\ If\ y=\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right),\ then\ \frac{d^2y}{dx^2}=$$

$a)\ 0$

$b)\ \sin 2x$

$c)\ \cos x$

$d)\ -\cos 2x$

$$Solution: y = \tan^{-1}\left[\frac{1-\tan x}{1+\tan x}\right] = \tan^{-1}\tan\left(\frac{\pi}{4}-x\right) = \left(\frac{\pi}{4}-x\right)$$

$$\Rightarrow y' = -1 \Rightarrow y'' = 0$$

$$Ans : a)$$

10.A point on the curve $y = 6x - x^2$ at which the tangent to the curve is included at an angle of 45^0 to the line x+y=0

a) (-3,9)   b)(-3,-27)

c) (3,9)   d)(0,0)

$$Solution: Slope.of.\tan gent = 6-2x;$$

$$Slope.of.the.given.line.is. -1$$

$$\therefore \tan 45^0 = \left|\frac{(6-2x)+1}{1+(6-2x)(-1)}\right|$$

$$\Rightarrow \frac{7-2x}{2x-5} = 1 \Rightarrow 4x = 12; x = 3, y = 9$$

$$Ans: (c)$$

11.In the curve $x^2y^2 = a^2(x^2 - a^2)$ if SN varies inversely as the nth power of the abscissa then n is equal to

a)n=2 b)n=3 c) n=4

d)n=1

$$Solutiion: y^2 = a^2 \frac{(x^2 - a^2)}{x^2} = a^2\left(1 - \frac{a^2}{x^2}\right)$$

$$2yy' = a^2(a^2 . \frac{2}{x^3}) \Rightarrow SN \alpha \frac{1}{x^3}$$

$$\Rightarrow n = 3$$

$$Ans: (b)$$

12. The surface area of a sphere, when its volume if increasing at the same rate as its radius is

a)1 b) $\frac{1}{2\sqrt{\pi}}$ c) 2 d) 4

$$Solution: V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow 1 = 4\pi r^2 \therefore \frac{dV}{dt} = \frac{dr}{dt}$$

$$S = 4\pi r^2 = 1 sq.units$$

$$Ans: (a)$$

13.Tangent is drawn to the ellipse $\frac{x^2}{27}+\frac{y^2}{1}=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ $\theta \in (0, \pi/2)$ Find the value of θ such that the sum of the intercepts on the coordinate axes by the tangent is minimum

a) pi/3 b) pi/6 c) pi/8 d) pi/4

$\pi/3$

$Solution: The.parametric.eqns.of.given.ellipse$

$x = 3\sqrt{3}\cos\theta, y = \sin\theta$

$Equation.of.\tan gent.at.\theta \Rightarrow \frac{x\cos\theta}{3\sqrt{3}} + y\sin\theta = 1$

$\Rightarrow \frac{x}{3\sqrt{3}\sec\theta} + \frac{y}{\cos ec\theta} = 1$

$f(\theta) = 0 \Rightarrow sum.of.\text{int}ercepts = 3\sqrt{3}\sec\theta + \cos ec\theta$

$f'(\theta) = 0 \Rightarrow 3\sqrt{3}\sec\theta\tan\theta - \cos ec\theta\cot\theta = 0$

$\Rightarrow \cot^3\theta = 3\sqrt{3} \Rightarrow \theta = \pi/6$

$Ans: (b)$

14. $\int (\sin^4 x - \cos^4 x)\,dx =$

a) $\frac{\cos 2x}{2} + c$

b) $\frac{-\sin 2x}{2} + c$

c) $\frac{\sin 2x}{2} + c$

d) $\frac{\cos 2x}{2} + c$

$$Solution: \int (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)dx$$

$$I = -\int \cos 2x dx = -\frac{\sin 2x}{2} + c$$

$$Ans: (b)$$

$$15.\int \frac{2x^2+3}{(x^2-1)(x^2+4)}\,dx$$

$$If.I = A\log\frac{x-1}{x+1} + B\tan^{-1}\frac{x}{2}.then.A\,\&\,B\,is$$

$a) -1,1$

$b) 1,-1$

$c) \frac{1}{2},\frac{1}{2}$

$d) -\frac{1}{2},\frac{1}{2}$

$$Solution: \int\left(\frac{1}{x^2-1}+\frac{1}{x^2+4}\right)dx = \frac{1}{2}\log\frac{x-1}{x+1}+\frac{1}{2}\tan^{-1}\frac{x}{2}$$

$Ans: c)$

$$16.\int \frac{a^{x/2}}{\sqrt{a^{-x}-a^{x}}}\,dx =$$

$$a)\ \frac{1}{\log a}\sin^{-1}(a^{x})+c$$

$$b)\ \frac{1}{\log a}\tan^{-1}(a^{x})+c$$

$$c)\ 2\sqrt{a^{x}-a^{-x}}+c$$

$$d)\ \log(a^{x}-a^{-x})+c$$

$$solution: \int \frac{a^{x/2}}{\frac{1}{a^{x/2}}\sqrt{1-a^{2x}}}dx = \frac{1}{\log a}\int \frac{a^x.\log a dx}{\sqrt{1-a^{2x}}}$$

$$put.t = a^x$$

$$\Rightarrow \frac{1}{\log a}\sin^{-1}(a^x)+c$$

$$Ans: a)$$

$$17. \int \frac{1}{1+\sin x+\cos x} dx =$$

$a) \log\left|1+\tan x/2\right|+c$

$b) \log\left|1+\sin x+\cos x\right|+c$

$c) 2\log\left|1+\tan x/2\right|+c$

$d) \frac{1}{2}\log\left|1+\tan x/2\right|+c$

$$Solution: I = \int \frac{dx}{(1+\cos x)+\sin x} = \int \frac{dx}{2\cos^2 x/2 + 2\sin x/2.\cos x/2}$$

$$\Rightarrow \int \frac{\sec^2 x/2.dx}{2(1+\tan x/2)} = \log\left|1+\tan x/2\right| + c$$

$Ans: a)$

$$18. \int 32x^3.(\log x)^2 dx =$$

$$a) 8x^4 (\log x)^2 + c$$

$$b) x^4.\left[8(\log x)^2 - 4\log x + 1 \right] + c$$

$$c) x^4.\left[8(\log x)^2 - 4\log x \right] + c$$

$$d) x^3.\left[(\log x)^2 + 2\log x \right] + c$$

Solution: Integration by parts.

$$\Rightarrow (\log x)^2 . \frac{32x^4}{4} - \int \frac{32x^4}{4} . \frac{2(\log x)}{x} dx = (\log x)^2 .8x^4 - \int 16x^3 . \log x dx$$

$$\Rightarrow (\log x)^2 .8x^4 - \left[(\log x). \frac{16x^4}{4} - \int \frac{16x^4}{4} . \frac{1}{x} . dx \right]$$

$$\Rightarrow (\log x)^2 .8x^4 - 4x^4 .(\log x) + 4x^4 + c$$

$$\Rightarrow x^4 \left[8(\log x)^2 - 4 \log x + 1 \right] + c$$

Ans: b)

$$19.\int_0^3 |2-x|\,dx =$$

$a)\ 5/2$

$b)\ 1/2$

$c)\ 7/2$

$d)\ 9$

$$Solution: We.know.that. \int |x| dx = \frac{1}{2} x|x|$$

$$\therefore \int_0^3 |2-x| dx = \left| \frac{1}{2}(x-2)|x-2| \right|_0^3$$

$$= \frac{1}{2}(1+4) = \frac{5}{2}$$

$$Ans: (a)$$

$$20. If\ .I_n = \int_0^{\pi/4} \tan^n x dx, then$$

$$\lim_{n\to\infty}\left[I_n + I_{n+2}\right] =$$

$a)1/2$

$b)1$

$c)\infty$

$d)0$

$$Solution: I_n = \int_0^{\pi/4} \tan^n x dx,$$

$$I_{n+2} = \int_0^{\pi/4} \tan^{n+2} x dx,$$

$$I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x(1+\tan^2 x)dx$$

$$= \int_0^{\pi/4} \tan^n x \sec^2 x dx = \left[\frac{\tan^{n+1} x}{n+1}\right]_0^{\pi/4} = \frac{1}{n+1}$$

$$\lim_{n\to\infty}\left[I_n + I_{n+2}\right] = 1$$

$$Ans: (b)$$

$$21.\ The\ value\ of \int_0^{\pi} \frac{dx}{5+3\cos x} =$$

$a)\ \pi/8$

$b)\ \pi/4$

$c)\ 0$

$d)\ \pi/2$

$$Solution: From.Direct.Observation$$

$$\int_0^{\pi} \frac{dx}{a+b\cos x} = \frac{\pi}{\sqrt{a^2-b^2}} (a^2 greater.than.b^2)$$

$$\int_0^{\pi} \frac{dx}{5+3\cos x} = \frac{\pi}{\sqrt{25-4}} = \frac{\pi}{4}$$

$$Ans: b)$$

22. The area between the curves $y = 2x - x^2$ and the $x - axis$ is

$a) 8/5$

$b) 4/3$

$c) 5/3$

$d) 7/3$

$$Solution : Now.y = 0 \Rightarrow 2x - x^2 = 0$$

$$simiplifing.we.get.x = 0, x = 2$$

$$A = \int_0^2 y dx = \int_0^2 (2x - x^2 dx$$

$$= \left[x^2 - \frac{x^3}{3} \right]_0^2 = 4 - 8/3 = 4/3$$

$$Ans : b)$$

23.*The.area.enclosed.between.the.curve.*

$y = \log_e(x+e)$ & *the.coordinate.axes.is*

$a)2$

$b)1$

$c)4$

$d)3$

$$Solution : clearly.y = \log_e (x+e).cuts.the.x - axis.at$$

$$(1-e,0)\ \&\ y - axis.at(0,1)$$

$$A = \int_{1-e}^{0} y dx = \int_{1-e}^{0} \log(x+e) dx$$

$$= \left(\log(x+e).x\right)_{1-e}^{0} - \int_{1-e}^{0} x.\frac{1}{x+e} dx$$

$$= 0 - \int_{1-e}^{0} \left(1 - \frac{e}{x-e}\right) dx = -\left[x - e\log(x+e)\right]_{1-e}^{0}$$

$$= \left[-e-(1-e)\right] = 1 : Ans : b)$$

$$23. If.\, f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + 4\sin x & \sin x & 1 \end{vmatrix}$$

$$then.the.value.of.\int_0^{\pi/2} f(x)dx =$$

$a)3$

$b)2/3$

$c)1/3$

$d))0$

$$Solution: f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4\sin x & 3 & 4\sin x \\ 1 + 4\sin x & \sin x & 1 \end{vmatrix}$$

$$Consider.C_1 \to C_1 - (C_2 + C_3)$$

$$f(x) = \begin{vmatrix} \sin x & \sin 2x & \sin 3x \\ 0 & 3 & 4\sin x \\ 0 & \sin x & 1 \end{vmatrix}$$

$$= Sinx(3 - 4\sin^2 x) = \sin 3x$$

$$\therefore \int_0^{\pi/2} \sin 3x dx = \left[\frac{-\cos 3x}{3}\right]_0^{\pi/2} = -1/3(0-1) = 1/3$$

$Ans: (c)$

24. The order and the degree of the differential equation

$$\left(1+3\frac{dy}{dx}\right)^{1/3}=4\frac{d^3y}{dx^3}\text{ are}$$

$a)(1, 2/3)$

$b)(3, 4)$

$c)(3, 3)$

$d)(1, 2)$

$$Solution: 1+3\frac{dy}{dx}=4^3\left(\frac{d^3y}{dx^3}\right)^3$$

$$Order = 3, Degree = 3$$

$$Ans: c)$$

25.The.differential.equation.of .all.non−vertical.lines.in.a.plane.is

$$a)\ \frac{d^2y}{dx^2}=0$$

$$b)\ \frac{dx}{dy}=0$$

$$c)\ \frac{dy}{dx}=0$$

$$d)\ \frac{d^2x}{dy^2}=0$$

$$Solution: The.differential.equation.of.all.non-vertical.lines.in.a.plane.is$$
$$given.by$$
$$ax+by=c, b\neq 0$$
$$a+b\frac{dy}{dx}=0 \Rightarrow b.\frac{d^2y}{dx^2}=0$$
$$\Rightarrow \frac{d^2y}{dx^2}=0$$
$$Ans: a)$$

26. The solution of the equation

$$\frac{d^2y}{dx^2} = e^{-2x} \text{ is } y =$$

a) $\frac{e^{-2x}}{4}$

b) $\frac{e^{-2x}}{4} + cx + d$

c) $\frac{e^{-2x}}{4} + cx^2 + d$

d) $\frac{e^{-2x}}{4} + c + d$

$$Solution : \frac{d^2 y}{dx^2} = e^{-2x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{2} e^{-2x} + c$$

$$\Rightarrow y = -\frac{1}{2}\left(-\frac{1}{2}\right) e^{-2x} + cx + d$$

$$\Rightarrow y = \frac{1}{4} e^{-2x} + cx + d$$

$Ans : b)$

CALCULAS
PART-B
MULTIPLE CHOICE QNS

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$01. If.\, f(x) = e^x, g(x) = \sin^{-1} x \,\&\, h(x) = f(g(x))$

$then. \dfrac{h'(x)}{h(x)} =$

$a) \sin^{-1} x$

$b) \dfrac{1}{\sqrt{1-x^2}}$

$c) \dfrac{1}{1-x^2}$

$d) e^{\sin^{-1} x}$

$$Solution: f(x) = e^x, g(x) = \sin^{-1} x$$

$$\Rightarrow h(x) = f(g(x))$$

$$\Rightarrow h(x) = f(\sin^{-1} x)$$

$$\Rightarrow h(x) = e^{\sin^{-1} x} \Rightarrow h'(x) = e^{\sin^{-1} x} . \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{h'(x)}{h(x)} = \frac{1}{\sqrt{1-x^2}}$$

$$Ans: b)$$

$02. If\ y = \sinh e^{x}, then$

$$\frac{dy}{dx} =$$

a) $\cosh e^{x}$

b) $-\cosh e^{x}$

c) $e^{x} \cosh e^{x}$

d) $\frac{1}{1+y^{2}}$

Answer:c)

$03. If\ f(x) = \cot^{-1}\left[(\cos 2x)^{1/2}\right] then$

$f'(\frac{\pi}{6}) =$

$a)\sqrt{\frac{2}{3}}$

$b)\sqrt{\frac{3}{2}}$

$c)2/3$

$d)3/2$

$$Solution:\ f(x)=\cot^{-1}\left[(\cos 2x)^{1/2}\right]$$

$$f'(x)=-\frac{1}{1+\cos 2x}.\frac{1}{2\sqrt{\cos 2x}}.(-\sin 2x).2$$

$$\Rightarrow f'\left(\frac{\pi}{6}\right)=-\frac{1}{1+\frac{1}{2}}.\frac{1}{2\sqrt{1/2}}.(\frac{-\sqrt{3}}{2}).2$$

$$\Rightarrow \frac{2}{3}.\frac{1}{\sqrt{2}}=\sqrt{\frac{2}{3}}$$

$Ans: a)$

$$04.\ If\ .y = \log \sin^2 \frac{x}{2},\ then.\ \frac{dy}{dx} =$$

$a)\ 2\cot\frac{x}{2}$

$b)\ \tan\frac{x}{2}$

$c)\ \cot\frac{x}{2}$

$d)\ \tan^2\frac{x}{2}$

$$Solution : y = \log \sin^2 \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{\sin^2 \frac{x}{2}}.2\sin\frac{x}{2}.\cos\frac{x}{2}.\frac{1}{2}$$

$$= \cot\frac{x}{2}$$

$Ans : c)$

$$05. The. derivative. of\ .\sin^{-1}\left[\frac{1-x}{1+x}\right] w.r.t \sqrt{x}$$

$$a) -\frac{1}{\sqrt{1-x^2}}$$

$$b) \frac{-2}{1+x}$$

$$c) -\frac{1}{\sqrt{1-x}}$$

$$d) \frac{1}{\sqrt{1-x}}$$

$$Solution: u = \sin^{-1}\left[\frac{1-x}{1+x}\right] \& \ v = \sqrt{x}$$

$$u = \sin^{-1}\left[\frac{1-x}{1+x}\right].Put.x = \tan^2\theta$$

$$\Rightarrow u = \sin^{-1}\left(\sin(\frac{\pi}{2} - 2\theta)\right) = \frac{\pi}{2} - 2\theta = -2\tan^{-1}\sqrt{x}$$

$$\Rightarrow \frac{du}{dx} = -\frac{2}{1+x}.\frac{1}{2\sqrt{x}} = -\frac{1}{\sqrt{x}(1+x)} \ \& \ \frac{dv}{dx} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{du}{dv} = -\frac{2}{1+x}$$

$Ans: b)$

$$06. If\ .y=\tan^{-1}\left[\frac{\log(\frac{e}{x^2})}{\log ex^2}\right]+\tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right], then \frac{d^{2y}}{dx^2}=$$

$a)1$

$b)-1$

$c)0$

$d)-1/2$

$$Solution: y=\tan^{-1}\left[\frac{\log(\frac{e}{x^2})}{\log ex^2}\right]+\tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$$

$$y=\tan^{-1}\left(\frac{1-\log x^2}{1+\log x^2}\right)+\tan^{-1}3+\tan^{-1}(\log x^2)$$

$$y=\tan^{-1}1-\tan^{-1}(\log x^2)+\tan^{-1}3+\tan^{-1}(\log x^2)$$

$$y=constant$$

$$y'=0\Rightarrow y''=0$$

$$Ans: c)$$

$07.The.\tan gent.to.the.curve.x^2 = 2y.at\left(1,1/2\right).makes.an$
$angle.with.the.x-axis$

$a)30^0$

$b)90^0$

$c)45^0$

$d)60^0$

$$Solution: x^2 = 2y \Rightarrow 2x = 2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,1/2)} = 1 = slope.of.\tan gent = \tan\theta$$

$$\Rightarrow \theta = 45^0$$

$$Ans: c)$$

08. The Curves $\frac{x^2}{16}+\frac{y^2}{25}=1$ & $\frac{x^2}{a}+\frac{y^2}{16}=1$ cut orthogonally,

then, $a=$

a) 6

b) 4

c) 7

d) 9

$$Solution: The.ellipses.\frac{x^2}{A}+\frac{y^2}{B}=1 \& \frac{x^2}{a}+\frac{y^2}{b}=1$$

$$cut.orthogonally, then.A-B=a-b$$

$$\Rightarrow 16-25=a-16$$

$$\Rightarrow a=32-25$$

$$\Rightarrow a=7$$

$$Ans: c)$$

$09.If.the.subnormal.at.any.point.on.the.curve$

$y^n = ax.is.a.constant.then.n =$

$a)2$

$b)1$

$c)3/2$

$d)-2$

$Solution: The.length.of.subnormal.at.any.point.on.the.curve$

$will.be.a.constant, if.only.the.curve.is.a.parabola.y^2 = 4ax$

$The.Curve.y^n = ax.will.be.of.form: y^2 = ax \Rightarrow n = 2$

$Ans: a)$

10.The.sides.of .an.equilateral.triangle.are.increasing.at.the.
rate.of .2cm / sec.The.rate.at.which.the.area.is.increases.when
the.side.is.10cm.is

$a)\sqrt{3} sq.units / \sec$

$b) 10 sq.units / \sec$

$c) 10\sqrt{3} sq.units / \sec$

$d) \frac{10}{\sqrt{3}} sq.units / \sec$

$$Solution: Let.l = length.of.the.sides.of.the.equilateral.triagnle$$

$$Area = A = \frac{\sqrt{3}}{4} l^2 \Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}.l.\frac{dl}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2}.l.2 = \sqrt{3}l$$

$$\Rightarrow \left(\frac{dA}{dt}\right)_{l=10} = 10\sqrt{3} sq.units$$

$$Ans: c)$$

$$11. The.Maximum.value.of.\frac{\log x}{x} =$$

$a)\ \frac{1}{2}\log 2$

$b)\ 0$

$c)\ 1/e$

$d)\ 1$

$$Solution : y = \frac{\log x}{x}$$

$$\frac{dy}{dx} = \log x.\left(-\frac{1}{x^2}\right) + \frac{1}{x}.\frac{1}{x} = -\frac{1}{x^2}(\log x - 1)$$

$$\frac{dy}{dx} = 0 \Rightarrow x = e \,\&\, \frac{d^2y}{dx^2} \angle 0$$

$$Max.Value = \frac{\log e}{e} = 1/e$$

$$Ans : c)$$

$$12. \int \frac{\cos 4x + 1}{\cot x - \tan x} dx = A \cos ecx + B, then$$

$a) A = -1/2$

$b) A = -1/8$

$c) A = -1/4$

$d) A = 1/4$

$$Solution: \int \frac{(\cos 4x+1)\cos x.\sin x}{\cos^2 x-\sin^2 x}dx$$

$$=\frac{1}{2}\int \frac{2\cos^2 2x.\sin 2x}{\cos 2x}dx=\int \sin 2x.\cos 2x.dx=\frac{1}{2}\int \sin 4x.dx=\frac{-1}{8}\cos 4x+c$$

$$Ans: b)$$

$$13. \int e^{3\log x} . (x^4 + 1)^{-1} . dx =$$

$$a) 1/4 \log(x^4 + 1) + c$$

$$b) -\log(x^4 + 1) + c$$

$$c) \log(x^4 + 1) + c$$

$$d) \frac{1}{x^4 + 1} + c$$

$$Solution: \int \frac{x^3}{x^4+1}dx = \frac{1}{4}\int \frac{4x^3}{x^4+1}dx = \frac{1}{4}\log(x^4+1)+c$$

$Ans: a)$

$$14. \int e^{4x} \sin 6x . \cos 2x dx =$$

$$a) \frac{e^{4x}(\sin 8x - 2\cos 8x)}{40} + \frac{e^{4x}(\sin 4x - \cos 4x)}{16} + c$$

$$b) \frac{e^{4x}(\sin 8x - 2\cos 8x)}{40} - \frac{e^{4x}(\sin 4x - \cos 4x)}{16} + c$$

$$c) -\frac{e^{4x}(\sin 8x - 2\cos 8x)}{40} + \frac{e^{4x}(\sin 4x - \cos 4x)}{16} + c$$

$$d) \frac{e^{4x}(\sin 8x - 2\cos 8x)}{40} - \frac{e^{4x}(\sin 4x - \cos 4x)}{16} + c$$

$$Solution: I=\frac{1}{2}\int e^{4x}(\sin 8x+\sin 4x)dx=\frac{1}{2}\int e^{4x}\sin 8x dx+\frac{1}{2}\int e^{4x}\sin 4x dx$$

$$\int e^{ax}\sin(bx+c)dx=\frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c)-b\cos(bx+c)\right]+c$$

$$using.we.get.: \frac{e^{4x}(\sin 8x-2\cos 8x)}{40}+\frac{e^{4x}(\sin 4x-\cos 4x)}{16}+c$$

$$15.\int \frac{e^x(1+x\log x)}{x}dx =$$

$$a)\ \frac{e^x \log x}{x} + c$$

$$b)\ e^x(1+\log x) + c$$

$$c)\ e^x \log x + c$$

$$d)\ xe^x \log x + c$$

$$Solution: I = \int \left(\log x + \frac{1}{x} \right) e^x dx = e^x \log x + c$$

$Ans: c)$

$$16. If\ .I_1 = \int \sin^{-1} x dx \ \& \ I_2 = \int \sin^{-1} \sqrt{1-x^2} dx, then$$

$a) I_1 = I_2$

$b) I_2 = \frac{\pi}{2} I_1$

$c) I_1 + I_2 = \frac{\pi}{2} x$

$d) I_1 + I_2 = \frac{\pi}{2}$

$$Solution: \sin^{-1}\sqrt{1-x^2} = \cos^{-1}x \Rightarrow I_1 + I_2 = \int(\sin^{-1}x + \cos^{-1}x)dx = \int\frac{\pi}{2}dx$$

$$I_1 + I_2 = \frac{\pi}{2}x$$

$$Ans: c)$$

$$17.\ \int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)} =$$

$a)\ \frac{\pi}{60}$

$b)\ \frac{\pi}{20}$

$c)\ \frac{\pi}{40}$

$d)\ \frac{\pi}{80}$

$$Solution : I = \frac{1}{5}\int_0^{\infty}\left(\frac{1}{x^2+4}-\frac{1}{x^2+9}\right)dx$$

$$= \frac{1}{5.2}\left[\tan^{-1}\frac{x}{2}\right]_0^{\infty} - \frac{1}{5.3}\left[\tan^{-1}\frac{x}{3}\right]_0^{\infty}$$

$$= \frac{1}{10}\left(\frac{\pi}{2}\right) - \frac{1}{15}\left(\frac{\pi}{2}\right) = \frac{\pi}{60}$$

$Ans : a)$

18. *Evaluate*

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx =$$

$a)\ \frac{\pi}{4}$

$b)\ \frac{\pi}{2}$

$c)$ *Zero*

$d)\ 1$

$$Solution : By.direct.observation$$

$$I = \frac{\pi}{4}$$

$$Ans : a)$$

19. $The.Area.of.the.region.bounded.by$

$a^2y^2 = x^2(a^2 - x^2).is:$

a) $\frac{a^2}{2} sq.unit$

b) $\frac{2a^2}{3} sq.unit$

c) $\frac{4a^2}{3} sq.unit$

d) $\frac{a^2}{4} sq.unit$

$$Solution: \text{Required}.Area.A = 4\int_0^a \frac{x}{a}\sqrt{a^2 - x^2}\,dx$$

$$= \frac{2}{a}\int_0^a 2x\sqrt{a^2 - x^2}\,dx$$

$$Put.a^2 - x^2 = t \Rightarrow 2xdx = -dt$$

$$A = -\frac{2}{a}\int_{a^2}^0 \sqrt{t}\,dt = -\frac{2}{a}.\frac{2}{3}[t^{3/2}]_{a^2}^0$$

$$A = \frac{4}{3}a^2 sq.unit.$$

$$Ans: c)$$

$20.The.Solution.of.the.equation.\frac{dy}{dx} = \cos(x - y)$

$is:$

$a) y + \cot\left(\frac{x - y}{2}\right) = c$

$b) x + \cot\left(\frac{x - y}{2}\right) = c$

$c) y + \tan\left(\frac{x - y}{2}\right) = c$

$d) x + \tan\left(\frac{x - y}{2}\right) = c$

$$Solution: \frac{dy}{dx} = \cos(x-y)$$

$$Put. x - y = t$$

$$\Rightarrow 1 - \frac{dt}{dx} = \frac{dy}{dx} \Rightarrow \frac{dt}{dx} = 1 - \cos t$$

$$\Rightarrow \int \frac{1}{1-\cos t} dt = \int dx \Rightarrow \int \frac{1}{2} \operatorname{cosec}^2 \frac{t}{2} = \int dx$$

$$\Rightarrow -\cot \frac{t}{2} = x + c$$

$$\Rightarrow x + \cot\left(\frac{x-y}{2}\right) = c$$

$$Ans: b)$$

ALL THE BEST

Vikasana - CET 2012