# CHAPTER:Indefinite Integrals
# Multiple Choice Questions with solution:

$01. \int \frac{1}{e^x - 1} dx =$

$a) \log(e^x - 1) - x + c$

$b) \log(1 - e^x) + c$

$c) \log(1 - e^x) + x + c$

$d) None$

$Ans : Put$

$e^x - 1 = t : e^x dx = dt \Rightarrow dx = \frac{dt}{(t+1)}$

$= \int \frac{1}{e^x - 1} dx = \int \frac{dt}{(t+1)(t)} = \int \left( \frac{1}{t} - \frac{1}{t+1} \right) dt$

$= \log(e^x - 1) - \log(e^x) + c$

$= \log(e^x - 1) - x + c$

$Ans : (a)$

$02. \int (\tan x + \cot x) dx =$

$a) \log \tan x + c$

$b) \log(\sin x + \cos x) + c$

$c) \log x + c$

$d) None$

$Solution :$

$I = \int \frac{(\tan^2 x + 1)}{\tan x} = \int \frac{\sec^2 x}{\tan x} dx$

$= \log(\tan x) + c$

$Ans : (a)$

$03.\int \sin(\log x)+\cos(\log x)dx =$

$a) x\sin(\log x)+c$

$b) x\cos(\log x)+c$

$c) \sin(\log x)-\cos(\log x)+c$

$d) none$

$Solution: Integration.by.parts$

$\Rightarrow \sin(\log x)x - \int x.\cos(\log x).\frac{1}{x}dx + \int \cos(\log x)dx$

$\Rightarrow \sin(\log x)x + c$

$Ans: a)$

$04.\int e^{-\log x}dx =$

$a) -e^{-\log x}+c$

$b) -xe^{-\log x}+c$

$c) \log x + c$

$d) xe^{-\log x}+c$

$Solution: \int e^{\log x^{-1}}dx = \int \frac{1}{x}dx = \log x + c$

$Ans: (c)$

$05.\int \left( \frac{\tan \frac{1}{x}}{x} \right)^2 dx =$

$a) x - \tan x + c$

$b) 1/x - \tan(1/x) + c$

$c) 1/x + \tan(1/x) + c$

$d) none$

$Solution: put$

$1/x = t \Rightarrow x = 1/t \Rightarrow dx = -1/t^2 dt$

$I = \int -\tan^2 t dt = \int (\sec^2 t - 1)dt$

$= t - \tan t + c$

$Ans: (b)$

$06.\int \frac{\tan\sqrt{x}}{\sqrt{x}}dx =$

$a)\log(\cos\sqrt{x}) + c$

$b)\log(\sec\sqrt{x}) + c$

$c)2\log(\sec\sqrt{x}) + c$

$d)none$

$Solution: put$

$t = \sqrt{x} \Rightarrow dt = 1/2\sqrt{x}dx$

$2dt = \frac{1}{\sqrt{x}}dx$

$I = 2\int \tan t dt = 2\log\sec\sqrt{x} + c$

$Ans: (b)$

$07.\int \frac{4x^3}{x^8+1}dx$

$a)\log(x^8+1) + c$

$b)4\tan^{-1}(x^4+1) + c$

$c)4\tan^{-1}\left(\frac{1}{4}x^4\right) + c$

$d)\tan^{-1}x^4 + c$

$Solution:$

$put$

$t = x^4 then$

$I = \tan^{-1}x^4 + c$

08..*If*

$$\int \frac{2^x}{\sqrt{1-4^x}}dx = k.\sin^{-1}(2^x), then$$

$k =$

$a)\log 2$

$b)1/2\log 2$

$c)(\log 2)^{-1}$

$d)1/2$

*Solution* :

$2^x = t$

$2^x.\log 2dx = dt$

$$I = \frac{1}{\log 2}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\log 2}\sin^{-1}(2^x) + c$$

*Ans* : ($c$)

$$09.\int \frac{\cos 2x}{(\sin x + \cos x)^2}dx =$$

$$a)\frac{-1}{\sin x + \cos x} + c$$

$b)\log(\sin x + \cos x) + c$

$c)\log(\sin x - \cos x) + c$

$d)\log(\sin + \cos x)^2 + c$

$$Solution: I = \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2}dx$$

$= \log(\sin x + \cos x) + c$

*Ans* : $b$)

$$10. \int \frac{1}{\sqrt{3-4x}}\,dx =$$

$a)\ \frac{1}{2\sqrt{3-4x}}+c$

$b)\ 2\sqrt{3-4x}+c$

$c)\ \frac{1}{\sqrt{3-4x}}+c$

$d)\ -\frac{1}{2\sqrt{3-4x}}+c$

$$Solution: I = \frac{-1}{4}\int \frac{-4}{\sqrt{3-4x}}\,dx = \frac{-1}{4}.2\sqrt{3-4x} = \frac{-1}{2}\sqrt{3-4x}+c$$

$Ans: d)$

$$11. \int \frac{(4x+3)}{(3x+7)}dx =$$

$a)\ \frac{4}{3}x$

$b)\ \frac{19}{9}\log(3x+7)+c$

$c)\ \frac{4x}{3}-\frac{19}{9}\log(3x+7)+c$

$d)\ \frac{4x}{3}-\log(3x+7)+c$

$$Solution: \int \frac{ax+b}{cx+d}dx = \frac{a}{c}x-\frac{(ad-bc)}{c^2}\log(cx+d)$$

$$I = \frac{4}{3}x-\frac{(28-9)}{9}\log(3x+7)+c$$

$$I = \frac{4}{3}x-\frac{19}{9}\log(3x+7)+c$$

$Ans: (c)$

$$12. \int x.2^x dx =$$

$$a) \frac{2^x}{\log 2}(x + \log x) + c$$

$$b) \frac{2^x}{\log 2}(\log \frac{e^x}{2}) + c$$

$$c) \frac{2^x}{(\log 2)^2}(x.\log 2 - 1) + c$$

$d)$ *none*

*Solution* : *Integration.by.parts.*

$$I = x.\frac{2^x}{\log 2} - 1.\frac{2^x}{(\log 2)^2} = \frac{2^x}{(\log 2)^2}(x.\log 2 - 1) + c$$

*Ans* : $(c)$

$$13. \int \frac{x^5}{x^2 + 1} dx =$$

$$a) \frac{x^4}{4} + \frac{x^2}{2} + \tan^{-1} x + c$$

$$b) \frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2}\log(x^2 + 1) + c$$

$$c) \frac{x^4}{4} + \frac{x^2}{2} - \tan^{-1} x + c$$

$$d) \frac{x^4}{4} - \frac{x^2}{2} - \tan^{-1} x + c$$

*Solution* : (*Division*)

$$I = \int (x^3 - x + \frac{x}{x^2 + 1}) dx$$

$$I = \frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2}\log(x^2 + 1) + c$$

*Ans* : $(b)$

$14.\int\left((x+1)^2\right)e^x dx$

$a) x.e^x + c$

$b) x^2 e^x + c$

$c) (x+1)e^x + c$

$d) (x^2+1)e^x + c$

$Solution: \int\left[\left((x^2+1)+2x\right)\right].e^x dx$

$I = (x+1)e^x + c$

$15.\int\frac{dx}{7+5\cos x} =$

$a) \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$

$b) \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{1}{\sqrt{6}}\tan\frac{x}{2}\right)+c$

$c) \frac{1}{7}\tan^{-1}(\tan x/2)+c$

$d) \frac{1}{4}\tan^{-1}(\tan x/2)+c$

$Solution: \int\frac{1}{a+b\cos x}dx = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\frac{\sqrt{a^2-b^2}}{a+b}\tan\frac{x}{2}\right]$

$I = \int\frac{1}{7+5\cos x}dx = \frac{2}{\sqrt{49-25}}\tan^{-1}\left[\frac{\sqrt{49-25}}{12}\tan\frac{x}{2}\right]+c$

$I = \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{1}{\sqrt{6}}\tan x/2\right)+c$

$Ans(b)$

$$16.\int (\sin^4 x-\cos^4 x)dx=$$

$a)\ \dfrac{\cos 2x}{2}+c$

$b)\ \dfrac{-\sin 2x}{2}+c$

$c)\ \dfrac{\sin 2x}{2}+c$

$d)\ \dfrac{\cos 2x}{2}+c$

$Solution:\int (\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)dx$

$I=-\int \cos 2x dx=-\dfrac{\sin 2x}{2}+c$

$Ans:(b)$

$$17.\int \frac{(a^x+b^x)^2}{a^x b^x}dx=$$

$a)\ \dfrac{(a/b)^x}{\log_e(a/b)}+\dfrac{(b/a)^x}{\log_e(b/a)}+2x+c$

$b)\ \dfrac{(a/b)^x}{\log_e(a/b)}-\dfrac{(b/a)^x}{\log_e(b/a)}+2x+c$

$c)(a/b)^x.\log(a/b)+(b/a)^x\log(b/a)+2x+c$

$d)(a/b)^x.\log(a/b)-(b/a)^x\log(b/a)-2x+c$

$Solution:$

$$\int \frac{(a^{2x}+b^{2x}+2a^x.b^x}{a^x.b^x}dx=\int (\frac{a^x}{b^x}+\frac{b^x}{a^x}+2)dx$$

$$=\frac{(a/b)^x}{\log_e(a/b)}+\frac{(b/a)^x}{\log_e(b/a)}+2x+c$$

$Ans:(a)$

$$18.\int \frac{\cos ec^2 x}{4+9\cot^2 x}dx =$$

$$a)-\frac{1}{3}\tan^{-1}\left(\frac{3\cot x}{2}\right)+c$$

$$b)-\frac{1}{6}\tan^{-1}\left(\frac{3\cot x}{4}\right)+c$$

$$c)\frac{1}{9}\log(4+9\cot^2 x)+c$$

$$d)-\frac{1}{6}\tan^{-1}\left(\frac{3\cot x}{2}\right)+c$$

*Solution* : *put*

$$3\cot x = t \Rightarrow -3\cos ec^2 x dx = dt$$

$$= \int \frac{-dt/3}{2^2+t^2} \Rightarrow \frac{-1}{6}\tan^{-1}\left(\frac{3\cot x}{2}\right)+c$$

*Ans* : $(d)$

19. *If*

$$\int \frac{dx}{5+4\cos x} = k.\tan^{-1}(M.\tan x/2)+c$$

*then*

$a) k = 1$

$b) k = 2/3$

$c) M = 4/3$

$d) M = 2/3$

$$Solution: I = \int \frac{dx}{5+4\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)} = \int \frac{1/2.\sec^2 x/2}{9+\tan^2 x/2}dx$$

$$\Rightarrow t = \tan x/2, dt = 1/2\sec^2 x/2dx$$

$$= 2\int \frac{dt}{t^2+3^2} = \frac{2}{3}\tan^{-1}\left(\frac{\tan x/2}{3}\right)+c$$

*Ans* : $k = 2/3 : (b)$

$$20.\int \cos ec^3 x dx =$$

$$a)\frac{1}{2}(-\cos ecx.\cot x)+\frac{1}{2}\log(\cos ecx+\cot x)+c$$

$$b)\frac{1}{2}(\cos ecx.\cot x)+\frac{1}{2}\log(\cos ecx+\cot x)+c$$

$$c)\frac{1}{2}(\cos ecx.\cot x)-\frac{1}{2}\log(\cos ecx+\cot x)+c$$

$$d)\frac{1}{2}(-\cos ecx.\cot x)-\frac{1}{2}\log(\cos ecx+\cot x)+c$$

*Solution*:

$$I=\int \cos ecx.\cos ec^2 x dx$$

$$I=-\cos ecx.\cot x-\int \cot^2 x.\cos ecx dx$$

$$I=-\cos ecx.\cot x-\int (\cos ec^2 x-1).co\sec x dx$$

$$I=-\cos ecx.\cot x-I+\log(\cos ecx+\cot x)+c$$

$$I=\frac{-\cos ecx.\cot x}{2}+\frac{\log(\cos ecx+\cot x)}{2}+c$$

*Ans*: $(a)$

$$21.\int\sqrt{\frac{x}{a^3-x^3}}dx = g(x)+c$$

*where*

$g(x)=$

$a)2/3\cos^{-1}x$

$b)2/3.\sin^{-1}\left(\frac{x^3}{a^3}\right)$

$c)2/3.\sin^{-1}\left(\sqrt{\frac{x^3}{a^3}}\right)$

$d)2/3.\cos^{-1}(x/a)$

*Solution* : *Put*

$$\left(\frac{x}{a}\right)^{3/2} = \sin\theta \Rightarrow \frac{1}{a}\frac{3}{2}\sqrt{\frac{x}{a}}$$

$$\Rightarrow \sqrt{x}dx = \frac{2}{3}a^{3/2}.\cos\theta d\theta$$

$$\Rightarrow I = \frac{2}{3}a^{3/2}\int\frac{\cos\theta d\theta}{a^{3/2}\sqrt{1-\sin^2\theta}} = \frac{2}{3}\sin^{-1}(\sin\theta)+c$$

$$=\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$$

*Ans* : $(c)$

$22. \int (\log x)^n dx$

*then*

$I_n + I_{n-1} =$

$a)(x.\log x)^n$

$b)x(\log x)^n$

$c)n.(\log x)$

$d)(\log x)^{n-1}$

*Solution* :

*Integration.by.parts.we.get*

$I_n = \int (\log x)^n .1.dx$

$= (\log x)^n .x - \int \frac{x.n(\log x)^{n-1} dx}{x}$

$I_n = (\log x)^n .x - n.I_{n-1}$

$\Rightarrow I_n + I_{n-1} = x.(\log x)^n$

*Ans* : $b)$

$23. \int \frac{\cos 4x + 1}{\cot x - \tan x} dx = A \cos ecx + B, then$

$a) A = -1/2$

$b) A = -1/8$

$c) A = -1/4$

$d) None$

*Solution* : $\int \frac{(\cos 4x + 1)\cos x.\sin x}{\cos^2 x - \sin^2 x} dx$

$= \frac{1}{2} \int \frac{2\cos^2 2x.\sin 2x}{\cos 2x} dx = \int \sin 2x.\cos 2x.dx = \frac{1}{2} \int \sin 4x.dx = \frac{-1}{8} \cos 4x + c$

*Ans* : $b)$

$24.\int \frac{2x^2+3}{(x^2-1)(x^2+3)}dx$

$If.I = A\log\frac{x-1}{x+1} + B\tan^{-1}\frac{x}{2}.then.A\,\&\,Bis$

$a) -1,1$

$b) 1,-1$

$c) \frac{1}{2},\frac{1}{2}$

$d) -\frac{1}{2},\frac{1}{2}$

$Solution: \int\left(\frac{1}{x^2-1}+\frac{1}{x^2+4}\right)dx = \frac{1}{2}\log\frac{x-1}{x+1} + \frac{1}{2}\tan^{-1}\frac{x}{2}$

$Ans: c)$

$25.\int 32x^3.(\log x)^2 dx =$

$a) 8x^4(\log x)^2 + c$

$b) x^4.\left[8(\log x)^2 - 4\log x + 1\right] + c$

$c) x^4.\left[8(\log x)^2 - 4\log x\right] + c$

$d) x^3.\left[(\log x)^2 + 2\log x\right] + c$

$Solution: Integration.by.parts.$

$\Rightarrow (\log x)^2.\frac{32x^4}{4} - \int \frac{32x^4}{4}.\frac{2(\log x)}{x}dx = (\log x)^2.8x^4 - \int 16x^3.\log x dx$

$\Rightarrow (\log x)^2.8x^4 - \left[(\log x).\frac{16x^4}{4} - \int \frac{16x^4}{4}.\frac{1}{x}.dx\right]$

$\Rightarrow (\log x)^2.8x^4 - 4x^4.(\log x) + 4x^4 + c$

$\Rightarrow x^4\left[8(\log x)^2 - 4\log x + 1\right] + c$

$Ans: b)$