6) (c) The potential energy of 'm' on the surface of earth is

$$U_1 = -\frac{GMm}{R}$$

Its potential energy at a height 'h' from the surface of earth is

$$U_2 = \frac{-GMm}{R+h}$$

Here $h = R$

$$\therefore U_2 = -\frac{GMm}{2R}$$

$\therefore$ Gain in potential energy is $\Delta U = U_2 - U_1$

$$= -\frac{GMm}{2R} + \frac{GMm}{R}$$

$$= \frac{1}{2}\frac{GMm}{R} = \frac{gR^2 \cdot m}{2R}$$

$$\boxed{\Delta U = \frac{1}{2} mgR}$$

7) (a) We have

$$V_o = \sqrt{gR} \qquad V_e = \sqrt{2gR}$$

To escape the satellite away from its orbit, change in velocity required is

$$\frac{V_e - V_o}{V_o} = \frac{\sqrt{2gR} - \sqrt{gR}}{\sqrt{gR}}$$

$$= \sqrt{2} - 1 = 0.414$$

$$= \underline{41.4\%}$$

8) (d) When earth revolves around the sun, its radius vector covers equal areas in equal interval of time according to Kepler's II law.

i.e. its areal velocity remains constant.

$\because$ earth doesn't move with constant speed around sun, its angular velocity & K.E change. P.E also changes the path is elliptical

9) Kepler's III law

$$T^2 \propto r^3$$

$$\frac{T_p^2}{T_e^2} = \frac{r_p^3}{r_e^3}$$

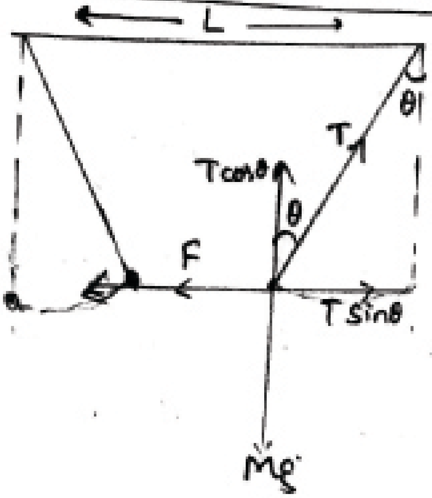
$$\frac{(27\, T_e)^2}{T_e^2} = \left(\frac{r_p}{r_e}\right)^3$$

$$\frac{r_p}{r_e} = (27)^{2/3}$$

$$\boxed{r_p = 9\, r_e}$$

SOLUTIONS

1) (c)



$$T\sin\theta = F = \frac{GM.M}{L^2}$$

$$T\sin\theta = \frac{GM^2}{L^2} \rightarrow ①$$

$$T\cos\theta = Mg \rightarrow ②$$

$$① \div ② \Rightarrow \tan\theta = \frac{GM}{gL^2}$$

$$\boxed{\theta = \tan^{-1}\left[\frac{GM}{gL^2}\right]}$$

2) (c) The work done in jumping against gravity is stored as potential energy.

The ability of the boy to jump remains same.

i.e. $mgh = mg'h'$

$$h' = \frac{g}{g'}.h$$

$$= \frac{g}{g/5}.2$$

$$\boxed{h' = 10m}$$

3) (a) If $h > R$, $g' = g\left(1 - \frac{2h}{R}\right)$ is not applicable.

$$\therefore g' = \frac{GM}{(R+h)^2}$$

$$\frac{g}{9} = \frac{gR^2}{(R+h)^2}$$

$$(R+h)^2 = 9R^2$$

$$R+h = 3R$$

$$\boxed{h = 2R}$$

4) (c) ∵ 'h' & 'd' are small,

$$g' = g\left(1 - \frac{2h}{R}\right) \qquad g' = g\left(1 - \frac{d}{R}\right)$$

$$g' = g - \frac{2hg}{R} \qquad g' = g - \frac{gd}{R}$$

$$g - g' = \frac{2hg}{R} \rightarrow ① \qquad g - g' = \frac{gd}{R} \rightarrow ②$$

Given ⇒ ① = ②

$$\therefore \frac{2hg}{R} = \frac{gd}{R}$$

$$\therefore \boxed{d = 2h}$$

5) (c) $g' = g - \omega^2 R\cos^2\lambda$

At poles $\lambda = 90°$

$$\therefore \boxed{g' = g}$$

∴ 'ω' doesn't affect the value of 'g' at poles.

Anywhere else, as 'ω' increases g' decreases.

WAVES

5. The wavelengths of two notes in air are $\frac{36}{195}$ m and $\frac{36}{193}$ m. Each note produces 10 beats per second separately with a third note of fixed frequency. The velocity of sound in air in ms^{-1} is

a) 330 b) 340 c) 350 d) 360

---

6. A 20cm long string, having a mass of 1.0g, is fixed at both the ends. The tension in the string is 0.5N. The string is set into vibration using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

a) 5 b) 6 c) 2 d) 3/2.

---

7. An organ pipe open at one end is vibrating in first overtone and is in resonance with another pipe open at both ends and vibrating in third harmonic. The ratio of length of two pipes is

a) 1 : 2 b) 4 : 1 c) 8 : 3 d) 3 : 8

---

8. A bat flies at a steady speed of $4 ms^{-1}$, emitting a sound of frequency $f = 90$ kHz. It is flying horizontally towards a vertical wall. The frequency of the reflected sound as detected by the bat will be (velocity of sound in air is $330 ms^{-1}$)

a) 88.1 kHz b) 87.1 kHz

c) 92.1 kHz d) 89.1 kHz.

WAVES

1. The ratio of speed of sound in nitrogen and that in helium gas at 300K is

a) $\sqrt{\frac{2}{7}}$ b) $\frac{\sqrt{1}}{7}$ c) $\frac{\sqrt{3}}{5}$ d) $\frac{\sqrt{6}}{5}$. ✓

---

2. Equation of progressive wave is

$$y = a \sin \left(10\pi x + 11\pi t + \pi/3\right)$$

a) its wavelength is 0.2 units

b) it is travelling in the positive x-direction

c) wave velocity is 1.5 units

d) time period of SHM is 1s. ✓

---

3. The phase difference between two points is $\pi/3$. If the frequency of wave is 50Hz, then what is the distance between two points.? (Given $V = 330ms^{-1}$)

a) 2.2m b) 1.1m c) 0.6m d) 1.7m

---

4. Two tuning forks P and Q when set vibrating, give 4 beats/s. If a prong of the fork P is filed, the beats are reduced to $2s^{-1}$. What is the frequency of P, if that of Q is 250 Hz?

a) 246 Hz b) 250 Hz c) 254 Hz d) 252 Hz

---

5. (c) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{3}}$

$\therefore \omega = \sqrt{3}$

$A = \frac{\text{path length}}{2} = \frac{4}{2}$

$A = 2\text{cm}.$

$a = v \qquad y = ?$

$\omega^2 y = \omega\sqrt{A^2 - y^2}$

$\omega y = \sqrt{A^2 - y^2}$

$\sqrt{3}\, y = \sqrt{A^2 - y^2}$

$A^2 - y^2 = 3y^2$

$A^2 = 4y^2 \Rightarrow A = 2y.$

$\therefore y = \frac{A}{2} \Rightarrow \boxed{y = 1\text{cm}}$

6. (a) We have $k_p = k_1 + k_2$ and $k_s = \frac{k_1 k_2}{k_1 + k_2}$

frequency of oscillation is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\frac{f_s}{f_p} = \sqrt{\frac{k_s}{k_p}}$

$= \sqrt{\frac{k_1 k_2}{k_1 + k_2} \times \frac{1}{(k_1 + k_2)}}$

$\because k_1 = k_2 = k$

$\frac{f_s}{f_p} = \sqrt{\frac{k^2}{4k^2}} = \frac{1}{2}$

$\therefore \boxed{\frac{f_s}{f_p} = \frac{1}{2}}$

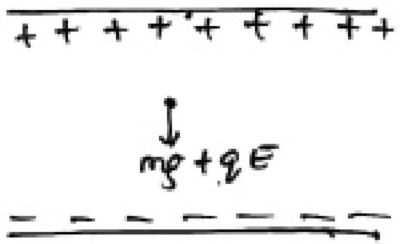
7. (a) $T = 2\pi\sqrt{l/g}.$

As the girl stands up, the centre of gravity of the pendulum shifts upwards and hence the length of the pendulum decreases

$\therefore$ 'T' decreases.

8. ~~When the lift ascends~~

(c) $T_0 = 2\pi\sqrt{l/g}$



when charged,

$F_{net} = mg + qE$

$g_{eff} = g + \frac{qE}{m}$

$\therefore T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

$\therefore T = 2\pi\sqrt{\frac{l}{\left(g + \frac{qE}{m}\right)}}$

$\therefore \frac{T}{T_0} = \sqrt{\frac{g}{g + \frac{qE}{m}}}$

$\therefore \boxed{\frac{T}{T_0} = \left(\frac{g}{g + \frac{qE}{m}}\right)^{1/2}}$

OSCILLATIONS.

1. (b) $y = \sin^2 \omega t$.

$\frac{dy}{dt} = 2\omega \sin\omega t \cdot \cos\omega t$

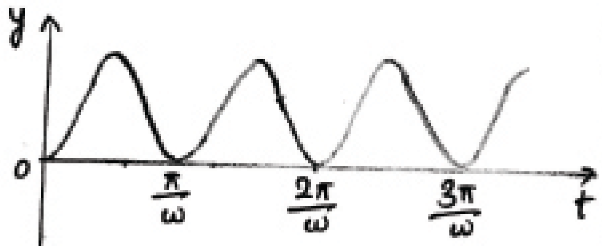
$v = \omega \sin 2\omega t$

$\frac{dv}{dt} = 2\omega^2 \cos 2\omega t = a$

For SHM $a \propto -y$

$\therefore y = \sin^2\omega t$ is not SHM.

But periodic



$\boxed{T = \frac{\pi}{\omega}}$

2. (b) $y_1 = 5\left[\sin 2\pi t + \sqrt{3}\cos 2\pi t\right]$

$= 10\left[\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right]$

$= 10\left[\cos\frac{\pi}{3}\cdot\sin 2\pi t + \sin\frac{\pi}{3}\cdot\cos 2\pi t\right]$

$y_1 = 10.\ \sin\left[2\pi t + \frac{\pi}{3}\right] \longrightarrow (1)$

$y_2 = 5\sin\left[2\pi t + \pi/4\right] \rightarrow (2)$.

$A_1 = 10$, $A_2 = 5$

$\boxed{\frac{A_1}{A_2} = \frac{2}{1}}$

3. (d) $x = A\sin\omega t$

$x = 1\sin\frac{2\pi}{T}.t$

$x = \sin\left(\frac{2\pi}{8}.t\right)$

$x = \sin\left(\frac{\pi t}{4}\right)$.

$a = -\omega^2 x$

$= -\left(\frac{2\pi}{T}\right)^2 \cdot \sin\left(\frac{\pi t}{4}\right)$

$t = \frac{4}{3}s$

$a = -\frac{4\pi^2}{64}\cdot\sin\left(\frac{\pi}{4}\cdot\frac{4}{3}\right)$

$= -\frac{\pi^2}{16}\times\frac{\sqrt{3}}{2}$

$\boxed{a = -\frac{\sqrt{3}}{32}\pi^2\ cms^{-2}}$

4. (c) $y = A\sin\omega t$.

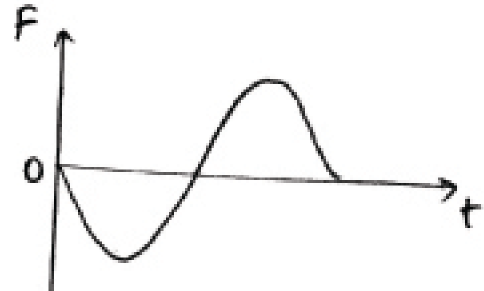
$\therefore$ acceleration is

$a = -\omega^2 A\sin\omega t$

$F = ma$

$\therefore F = -m\omega^2 A\sin\omega t$

$\therefore \boxed{F \propto -\sin\omega t}$



4) The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct?

a) $d = \frac{h}{2}$ b) $d = \frac{3h}{2}$ c) $d = 2h$ d) $d = h$

5) If the earth were to spin faster, acceleration due to gravity at the poles

a) increases
b) decreases
c) remains the same
d) depends on how fast it spins.

6) If 'g' is the acceleration due to gravity on the
X earth's surface, the gain of the potential energy of an object of mass 'm' raised from the surface of the earth to a height equal to radius R of the earth is

a) $2mgR$ b) mgR c) $\frac{1}{2}mgR$ d) $\frac{1}{4}mgR$

7) A satellite is orbiting around the earth. By what percentage should we increase its velocity so as to enable it escape away from the earth?

a) 41.4% b) 50% c) 82.8% d) 100%

6. (a) Distance between successive nodes is $d = \frac{\lambda}{2}$.

$$d = \frac{v}{2f} \qquad \because \lambda = \frac{v}{f}$$

$$d = \frac{\sqrt{T/m}}{2f} \qquad v = \sqrt{T/m}$$

$$m = \frac{M}{l} = \frac{1g}{20cm}$$

$$d = \frac{\sqrt{0.5/5\times10^{-3}}}{2\times100} \qquad m = 5\times10^{-3}\,kgm^{-1}$$

$$d = \frac{10}{200} = \frac{1}{20} = 0.05\,m$$

$$\boxed{d = 5cm}$$

7. (a) Frequency of first overtone in closed pipe is

$$f_1 = \frac{3V}{4l_1} \longrightarrow (1)$$

Frequency of third hormonic of open pipe is

$$f_2 = \frac{3V}{2l_2} \longrightarrow (2)$$

For resonance, $f_1 = f_2$

$$\therefore \frac{3V}{4l_1} = \frac{3V}{2l_2}$$

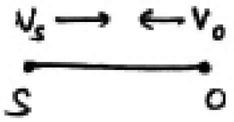
$$l_2 = 2l_1$$

$$\boxed{l_1 : l_2 = 1:2}$$

8. Doppler effect,
(c)

$$f' = f\left(\frac{V \pm V_0}{V \pm V_s}\right)$$

Here source and observer [both are the bat] are approaching each other.

$\therefore$ 

V_s is $-ve$ & V_0 is $+ve$.

$$\therefore f' = f\left(\frac{V + V_0}{V - V_s}\right)$$

$$f' = 90\times10^{3}\left(\frac{330+4}{330-4}\right)$$

$$\boxed{f' = 92.1\,kHz}$$

WAVES

SOLUTIONS

1. (c) $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{M}} = \sqrt{\frac{\gamma RT}{M}}$

$M \rightarrow$ Molecular weight.

For mono atomic gas Helium,

$\gamma_1 = \frac{5}{3}$, $M_1 = 4$

For diatomic gas nitrogen

$\gamma_2 = \frac{7}{5}$, $M_2 = 28$.

$\therefore \frac{V_{N_2}}{V_{He}} = \sqrt{\frac{\gamma_2}{\gamma_1} \times \frac{M_1}{M_2}}$

$= \sqrt{\frac{7/5}{5/3} \times \frac{4}{28}}$

$\frac{V_{N_2}}{V_{He}} = \sqrt{\frac{21}{25} \times \frac{1}{7}} = \sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{5}$

$\boxed{\frac{V_{N_2}}{V_{He}} = \frac{\sqrt{3}}{5}}$

2. (a) The standard eqn. is

$y = a \sin(kx + \omega t + \phi)$

Given $y = a \sin(10\pi x + 11\pi t + \pi/3)$

Comparing,

$k = 10\pi$ $\qquad \omega = 11\pi$

$\frac{2\pi}{\lambda} = 10\pi$ $\qquad \frac{2\pi}{T} = 11\pi$

$\boxed{\lambda = 0.2 \text{ units}}$ $\qquad T = \frac{2}{11}$ unit

$f = \frac{11}{2}$ units

$v = f\lambda = \frac{11}{2} \times 0.2$

3. (b) $\Delta x = \frac{\lambda}{2\pi} \cdot \Delta\phi$

$= \frac{v}{2\pi f} \cdot \Delta\phi$

$= \frac{330}{2\pi . 50} \times \frac{\pi}{3}$

$\boxed{\Delta x = 1.1 \text{ m}}$

4. (a) $Q \rightarrow 250$ Hz

$f_b = f_1 \sim f_2$

$\therefore P \rightarrow 246$ Hz or 254 Hz

On filing, frequency increases

$\therefore$ If $P \rightarrow 254$,

on filing $f_b > 4$ Bps

But given $f_b = 2$ Bps

$\therefore \boxed{P \rightarrow 246 \text{ Hz}}$

5) (d) Beat frequency is

$f_b = f_1 \sim f_2$

Let the frequency of 3rd note is f

$\frac{195v}{36} - f = 10 \rightarrow (1)$

$f - \frac{193v}{36} = 10 \rightarrow (2)$

(1) + (2) gives

$\frac{2v}{36} = 20$

$\boxed{V = 360 \text{ ms}^{-1}}$

GRAVITATION.

29/3/2016.

1) Two metallic spheres each of mass M are suspended by two strings each of length L. The distance b/w the upper ends of strings is L. The angles which the strings will make with the vertical due to mutual attraction of the spheres is

a) $\tan^{-1}\left[\frac{GM}{gL}\right]$ b) $\tan^{-1}\left[\frac{GM}{2gL}\right]$

c) $\tan^{-1}\left[\frac{GM}{gL^2}\right]$ d) $\tan^{-1}\left[\frac{2GM}{gL^2}\right]$

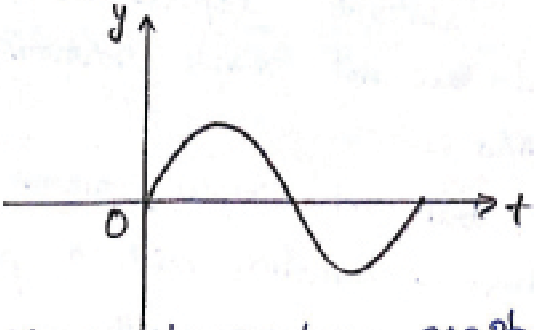
2) A boy can jump 2m on the surface of earth. How much can he jump on the surface of moon assuming that the acceleration due to gravity on the surface of moon is $\frac{1}{5}$th that on the surface of earth.

a) 2m b) 5m c) 10m d) $\frac{2}{5}$m

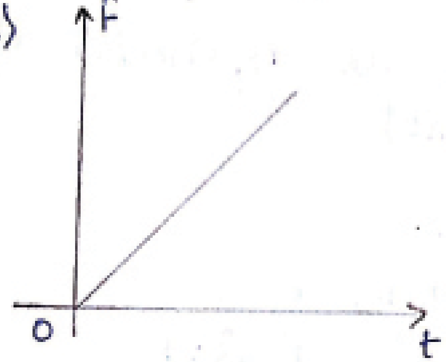
3) The height at which the acceleration due to gravity becomes $g/9$ [where g = accn due to gravity on the surface of earth] in terms of R, the radius of earth, is

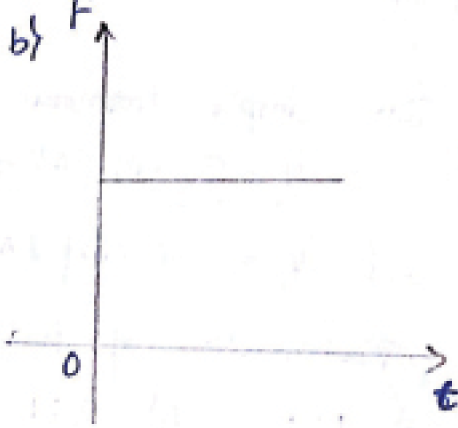
a) 2R b) $\frac{R}{\sqrt{3}}$ c) $\frac{R}{2}$ d) $\sqrt{2}$ R

4. The displacement - time graph of a particle executing SHM is as shown in the figure.

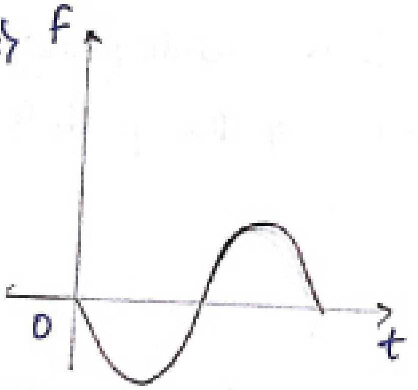


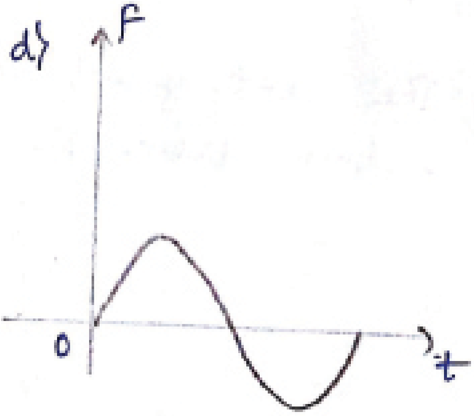
The corresponding force - time graph of the particle is

a) 

b) 

Short cut.

c) 

d) 

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5. A particle performing SHM has time period $\frac{2\pi}{\sqrt{3}}$ and path length 4 cm. The displacement from mean position at which acceleration is equal to velocity is

a) zero b) 0.5 cm c) 1 cm d) 1.5 cm

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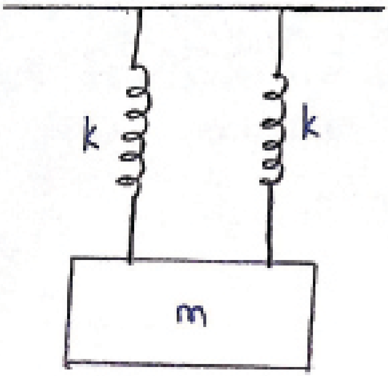
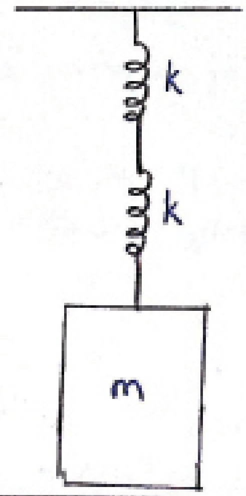
6. Two identical springs are connected in series and parallel as shown in the figure. If f_s and f_p are frequencies of series and parallel arrangements, then what is $f_s : f_p$?





a) 1:2 b) 2:1 c) 1:3 d) 3:1.

---

7. A girl swings on a cradle in a sitting position. If she stands what happens to the time period of girl and cradle?

a) Time period decreases
b) Time period increases
c) Remains constant
d) First increases and then decreases

Read out

---

8. A small sphere carrying a charge q is hanging in between two parallel plates by a string of length L. Time period of the pendulum is T_0. When the parallel plates are charged, the time period changes to T. The ratio T/T_0 is equal to

a) $\left[\dfrac{g + \frac{qE}{m}}{g}\right]^{1/2}$

b) $\left[\dfrac{g}{g + \frac{qE}{m}}\right]^{3/2}$

c) $\left[\dfrac{g}{g + \frac{qE}{m}}\right]^{1/2}$

d) None of these.

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b) $\left[\dfrac{g}{g + \frac{qE}{m}}\right]^{3/2}$

c) $\left[\dfrac{g}{g + \frac{qE}{m}}\right]^{1/2}$

d) None of these.

8) When earth moves around the sun, the quantity which remains constant is

a) angular velocity b) kinetic energy
c) potential energy d) areal velocity.

9) The period of a planet around sun is 27 times that of earth. The ratio of radius of planet's orbit to the radius of earth's orbit is

X

a) 4 b) 9 c) 64 d) 27.